Week 15
MTH-1322 – Calculus 2

Hello and Welcome to the weekly resources for MTH-1322 – Calculus 2!

This week is Week 15 of class, the last week of the semester, and below is a summary recap view of all the major topics covered throughout the course. You can find more detail on each topic in the earlier weekly resources listed on our website.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Keywords: *(In)definite Integral, Integrand, U-Substitution, Integration by Parts, Trig Identities, Solid of Revolution, Cross Section, Density, Partial Fraction Decomposition, L'Hopital's Rule, Improper Integral, Limit, Convergence, Divergence, Force, Mass, Acceleration, Work, Energy, Arc Length, Surface Area, Differential Equation, Particular Solution, General Solution, Initial Condition, Separation of Variables, Exponential Growth/Decay, Sequence, Convergence, Divergence, Squeeze Theorem, Bounded, Monotonic, Partial Sum, Linearity, Geometric Series, P-Series, Comparison Test, Algebraic Limit Laws, Absolute Convergence, Conditional Convergence, Alternating Series, Ratio Test, Root Test, Power Series, Center, Interval of Convergence, Radius of Convergence, Geometric Series, Term-by-Term, Power Series Expansion, Approximation, Agreement to Order n, Taylor Polynomial, Maclaurin Polynomial, Error Bound, Taylor Series Expansion, Maclaurin Series, Binomial Series, Euler's Formula*

Topic of the Week: Semester Review

Contents:

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Highlight 1: Integration by Substitution and Parts

There are basically two methods of integration: *substitution* and *parts*.

When integrating by *substitution*, look to see if the derivative of any one component of the *integrand* occurs somewhere else in the *integrand* as a factor. Call this component "u," and calculate the derivative of u, so you can *substitute* the dx for a term involving du. Proceed to integrate in terms of u, switching everything back to x at the end.

When integrating by *parts*, use the formula: $\int udv = uv - \int vdu$. The first step is to select u and dv from the given *integrand*. Typically, u will be a function that gets simpler when differentiated, although this is not always the case. dv is simply whatever is left over, including the dx factor. From u, calculate du by differentiating. From dv, calculate v by integrating. Apply the above *integration by parts* formula. Hopefully, the resultant integral will be immediately integrable. It is common that the resultant integral will not be immediately integrable, and you will need to apply *integration by parts* again.

Highlight 2: Trigonometric Integration

Basic forms: $\int sin^m x * cos^n x dx$ and $\int sec^m x * tan^n x dx$. The idea is to apply a *trig identity* to manipulate the *integrand* to a point where you can use *U-substitution* or *integration by parts*. The most useful identities are the following *Pythagorean identities*:

$$sin^2 x + cos^2 x = 1$$
$$sec^2 x = tan^2 x + 1$$
$$csc^2 x = 1 + cot^2 x$$

Highlight 3: Trigonometric Substitution

There are three basic forms integrands can take, such that a *trigonometric substitution* will prove useful when integrating:

$$\frac{1}{\sqrt{a^2 - x^2}}, \quad \frac{1}{x\sqrt{x^2 - a^2}}, \quad \frac{1}{\sqrt{x^2 + a^2}}$$

When the *integrand* assumes one of these forms, the trick will be to choose the *substitution* $u = c * trig(x)$, where c is some constant, specially chosen to allow for the formation of one of the above *Pythagorean trig identities* underneath the radical. When the *Pythagorean identity* is applied, one ends up with the square root of the square of a trig function, which, of course, just simplifies to a single trig function outside the radical, which is the whole point. Hopefully, the resultant *integrand* will be easier to integrate, and one can simply integrate with respect to u, switching back to x after performing the integration.

Highlight 4: Volume, Density, Mass, Average Value

$$Average\ Value = \frac{1}{b-a}\int_a^b f(x)dx$$

$$Mass = \int_a^b p(x)dx \ , \quad where\ p(x)\ is\ the\ density\ at\ point\ x.$$

$Volume = \int_a^b A(x)dx$, where A(x) is the *cross-sectional area* at height x.

Highlight 5: Disk and Shell Methods

Disk Method:

$$V = \pi\int_a^b (R^2{}_{outer} - R^2{}_{inner})dx = \pi\int_a^b (f(x)^2 - g(x)^2)dx$$

(Rogawski 377)

Shell Method:

$$V = 2\pi\int_a^b (radius)(height\ of\ shell)dx = 2\pi\int_a^b x\,(f(x) - g(x))dx$$

(Rogawski 387)

Highlight 6: Partial Fractions

There are 5 basic forms the denominator of *integrands* requiring *partial fraction decompositions* can take (Rogawski 426). They are as follows:

1. Non-repeated linear factors

$$\frac{1}{(x-2)(x-5)} = \frac{A}{x-2} + \frac{B}{x-5}$$

2. Repeated linear factors

$$\frac{3x-9}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$$

3. Irreducible quadratic factors

$$\frac{18}{(x+3)(x^2+9)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+9}$$

4. Reducible quadratic factors

$$\frac{18}{(x+3)(x^2-9)} = \frac{A}{x-3} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

OR

$$\frac{18}{(x+4)(x^2-9)} = \frac{A}{x+4} + \frac{B}{x-3} + \frac{C}{x+3}$$

5. Repeated quadratic factors

$$\frac{4-x}{x(x^2+2)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2}$$

If the degree of the numerator is greater than or equal to the degree of the denominator, it will be necessary to do *long division* prior to attempting a *partial fractions decomposition*.

Highlight 7: L'Hopitals Rule and Improper Integrals

The indeterminate forms of a limit are as follows:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 * \infty, \frac{1}{0 * \infty}$$

When a limit is in an indeterminate form, one can use *L'Hopital's Rule* to simplify the limit expression before evaluating. *L'Hopital's Rule* states the following:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

L'Hopital's Rule is sometimes useful in evaluating the limit expressions resulting from *improper integrals. Improper integrals* arise in two cases, in which one either wants to find the entire area under a curve along a horizontal asymptote, or beneath a vertical asymptote (Rogawski 442). For the first case, the *improper integral* is defined as follows:

$$\int_a^\infty f(x)dx = \lim_{R\to\infty} \int_a^R f(x)dx$$

For the second case, if there is a vertical asymptote at "a", the *improper integral* is defined,

$$\int_a^b f(x)dx = \lim_{R\to a^+} \int_a^R f(x)dx$$

And if there is a vertical asymptote at b, the *improper integral* is defined,

$$\int_a^b f(x)dx = \lim_{R\to b^-} \int_a^R f(x)dx$$

These definitions reduce the problem of finding an *improper integral* to two simpler problems: finding the antiderivative, and finding the limit of the antiderivative.

Formulas can be derived to provide rules for integrating special infinite forms like $\frac{1}{x^p}$. These formulas are useful for proving *convergence/divergence* of more complicated functions by means of a *comparison* (Rogawski 443, 447). The rules for integrating $\frac{1}{x^p}$ are as follows:

$$\int_a^\infty \frac{dx}{x^p} = \begin{Bmatrix} \frac{a^{1-p}}{p-1} & if\ p > 1 \\ diverges & if\ p \le 1 \end{Bmatrix}$$

$$\int_0^a \frac{dx}{x^p} = \begin{cases} \frac{a^{1-p}}{p-1} & if\ p < 1 \\ diverges & if\ p \geq 1 \end{cases}$$

Highlight 8: Work and Energy

Work, or *energy*, is *force* times distance. When *force* is variable across distance,

$$W = \int_a^b F(x)dx$$

In this formula, F is *force* and x is distance. The units of *work* are *Joules*, or $\frac{kg*m^2}{s^2}$.

Highlight 9: Arc Length and Surface Area

The formula for *arc length* is as follows:

$$S = \int_a^b \sqrt{1+f'(x)^2}dx$$

Similarly, the formula for *surface area* is as follows:

$$SA = 2\pi \int_a^b f(x)\sqrt{1+f'(x)^2}dx$$

(Rogawski 474)

Highlight 10: Differential Equations

A *differential equation* is an equation relating a function to one or more of its derivatives. A *particular solution* to a *differential equation* is a single function that satisfies the equation. A *general solution* is the form of every function that satisfies the equation. A *differential equation* is called "*separable*" if it can be manipulating into the following form:

$$\frac{dy}{dx} = f(x)g(y)$$

If a *differential equation* can be expressed in this form, it is a simple matter to multiply both sides by $\frac{dx}{g(y)}$ to get $\frac{dy}{g(y)} = f(x)dx$. From here, one need simply integrate both sides $\int \frac{dx}{g(y)} = \int f(x)dx$ and solve the resultant expression for y in terms of x. If solving for y is possible, y will be the *general solution* of the *differential equation*.

Exponential growth or *decay* happens when the rate of growth of a population is proportional to its size. The *differential equation* matching this description is as follows:

$$\frac{dy}{dt} = ky$$

Only the exponential function y = e^x is proportional to its own derivative, so the *general solution* to this *differential equation* will be $y(t) = De^{kt}$ (Rogawski 503). The form of this *solution* can be used to model lots of different natural phenomena involving *exponential growth* or *decay*.

Highlight 11: Sequences and Series

Loosely, a *sequence* is an infinitely long list of numbers. A *sequence converges* to L if $\lim_{n\to\infty} a_n = L$. If no such L exists, the *sequence diverges*. The *Algebraic Limit Laws* are given as follows:

THEOREM 2 Limit Laws for Sequences Assume that $\{a_n\}$ and $\{b_n\}$ are convergent sequences with

$$\lim_{n\to\infty} a_n = L, \qquad \lim_{n\to\infty} b_n = M$$

Then

(i) $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n = L \pm M$

(ii) $\lim_{n\to\infty} a_n b_n = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right) = LM$

(iii) $\lim_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} = \dfrac{L}{M}$ if $M \neq 0$

(iv) $\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n = cL$ for any constant c

(Rogawski 541).

A *sequence* is said to be *bounded from above* if there is some single number that is greater than every number in the *sequence*. A *sequence* is said to be *bounded from below* if there is some single number that is less than every number in the *sequence*. A *sequence* is simply called *bounded* if it is *bounded from above* and *from below*, and a *sequence* is called *unbounded* if it is not *bounded*.

A *monotonic increasing sequence* is one whose terms are always increasing. A *monotonic decreasing sequence* is one who's terms are always decreasing.

If $\{a_n\}$ is *bounded above* by M and is *increasing monotonic*, then $\{a_n\}$ *converges* and $\lim_{n\to\infty} a_n \leq M$. If $\{a_n\}$ is *bounded below* by m and is *decreasing monotonic*, then $\{a_n\}$ *converges* and $\lim_{n\to\infty} a_n \geq m$ (Rogawski 544).

A *series* is the sum of a *sequence*. The n^{th} *partial sum* (S_n) of an *infinite series* is the sum of the first n terms. A *series converges* if the *sequence* of *partial sums converges* to some L. *Convergent*, infinite *series* satisfy the *linearity property*,

THEOREM 1 Linearity of Infinite Series If $\sum a_n$ and $\sum b_n$ converge, then $\sum(a_n + b_n)$, $\sum(a_n - b_n)$, and $\sum ca_n$ also converge, the latter for any constant c. Furthermore,

$$\sum(a_n + b_n) = \sum a_n + \sum b_n$$

$$\sum(a_n - b_n) = \sum a_n - \sum b_n$$

$$\sum ca_n = c\sum a_n \qquad (c \text{ any constant})$$

(Rogawski 551).

For the special *geometric series* $\sum_{n=0}^{\infty} c * r^n$, the *nth partial sum* is as follows:

$$S_N = c + cr + cr^2 + cr^3 + \ldots + cr^N = \frac{c(1-r^{N+1})}{1-r}$$

And the entire *series* (infinite sum):

$$\sum_{n=0}^{\infty} c * r^n = c + cr + cr^2 + cr^3 + \ldots = \frac{c}{1-r}, provided \ -1 < r < 1.$$

For the infinite *geometric series*, if $r \geq 1$, then the *series diverges*.

Highlight 12: Convergence

There are a variety of *tests* to determine the *convergence/divergence* of infinite *series*.

THEOREM 1 Partial Sum Theorem for Positive Series If $\sum_{n=1}^{\infty} a_n$ is a positive series, then either

(i) The partial sums S_N are bounded above. In this case, $\sum_{n=1}^{\infty} a_n$ converges. Or,

(ii) The partial sums S_N are not bounded above. In this case, $\sum_{n=1}^{\infty} a_n$ diverges.

(Rogawski 561)

THEOREM 2 Integral Test Let $a_n = f(n)$, where f is a positive, decreasing, and continuous function of x for $x \geq 1$.

(i) If $\int_1^\infty f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

(ii) If $\int_1^\infty f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

(Rogawski 561)

THEOREM 3 Convergence of p-Series The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise.

(Rogawski 562)

THEOREM 4 Direct Comparison Test
Assume that there exists $M > 0$ such that $0 \leq a_n \leq b_n$ for $n \geq M$.

(i) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ also diverges.

(Rogawski 563)

THEOREM 5 Limit Comparison Test Let $\{a_n\}$ and $\{b_n\}$ be *positive* sequences. Assume that the following limit exists:

$$L = \lim_{n \to \infty} \frac{a_n}{b_n}$$

- If $L > 0$, then $\sum a_n$ converges if and only if $\sum b_n$ converges.
- If $L = \infty$ and $\sum a_n$ converges, then $\sum b_n$ converges.
- If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

(Rogawski 564)

In addition, the *Ratio Test* states,

$$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

$$If\ \rho < 1, then \sum_{n=1}^{\infty} a_n\ converges\ absolutely.$$

$$If\ \rho > 1, then \sum_{n=1}^{\infty} a_n\ diverges.$$

And the *Root Test* states,

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

$$If\ L < 1, then \sum_{n=1}^{\infty} a_n\ converges\ absolutely.$$

$$If\ L > 1, then \sum_{n=1}^{\infty} a_n\ diverges.$$

Recall the following vocabulary. $\sum_{n=1}^{\infty} a_n$ *converges absolutely* if $\sum_{n=1}^{\infty} |a_n|$ *converges*. $\sum_{n=1}^{\infty} a_n$ *converges conditionally* if $\sum_{n=1}^{\infty} a_n$ *converges* but $\sum_{n=1}^{\infty} |a_n|$ *diverges* (Rogawski 570). The *alternating series test* states that for any positive decreasing sequence $\{b_n\}$ that *converges* to zero, $\left(b_1 > b_2 > b_3 > \ldots > 0,\ \ \lim_{n\to\infty} b_n = 0\right)$, the following series *converges*:

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots$$

Highlight 13: Power Series

A *power series* is a *series* of the following form:

$$F(x) = \sum_{n=1}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$$

"c" is called the "*center*" of the *power series*, and a *power series converges* for values of x within a "*radius of convergence*" around c. One can discover the *radius of convergence* for a *power series* using the *convergence theorems* and techniques learned in previous sections.

Some functions have *power series* representations. The ability to substitute one for the other is useful because it can translate the problem of differentiating a complicated function into the problem of differentiating a polynomial, which is always straightforward. The concept of *term-by-term* differentiation (and integration) is spelled out below.

THEOREM 2 Term-by-Term Differentiation and Integration Assume that

$$F(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$$

has radius of convergence $R > 0$. Then F is differentiable on $(c-R, c+R)$. Furthermore, we can integrate and differentiate term by term. For $x \in (c-R, c+R)$,

$$F'(x) = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}$$

$$\int F(x)\,dx = A + \sum_{n=0}^{\infty} \frac{a_n}{n+1}(x-c)^{n+1} \qquad (A \text{ any constant})$$

For both the derivative series and the integral series the radius of convergence is also R.

(Rogawski 584)

Through the formula for the infinite sum of a *geometric series*,

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad for\ |x| < 1$$

and some algebraic manipulation, one can often figure out a *power series expansion* for many sorts of rational expressions, by getting them into the form $\frac{1}{1-u}$, where u is in terms of the underlying variable x. From there, one can apply the reverse of the above formula, and proceed to integrate *term-by-term*, if required.

The *power series expansion* of the exponential function is as follows:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

Highlight 14: Taylor Polynomials and Series

A *Taylor polynomial* is a function of the following form:

$$T_n(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

A *Taylor polynomial* is a limited approximation of the underlying function f around the *center* "a." A *Taylor polynomial agrees with f to the order n*. In other words, the more derivatives used in constructing the polynomial, the more accurate the approximation. A *Taylor polynomial* is a *Maclaurin polynomial* when *centered* at 0.

The *Error Bound Formula* gives a way of putting a cap on the uncertainty of a *Taylor polynomial approximation* of a given *order*. The formula states that if we know of a K such that $\left|f^{(n+1)}(u)\right| \le K,\ \forall\, u \in [a, x]$, then we can put the following *bound on the error* (Rogawski 596):

$$|f(x) - T_n(x)| \le K \frac{|x-a|^{n+1}}{(n+1)!}$$

A *Taylor series*, loosely speaking, is a *Taylor polynomial* where n = ∞. In other words,

$$T(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \ldots = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x - c)^n$$

So long as a function is infinitely differentiable at some point, it has a *Taylor series expansion* at that point. This principle is extremely useful, because most of the functions we deal with in lower math fall into this category.

As with *power series*, we can integrate a complicated underlying function simply by integrating its *power series expansion term-by-term*. One can combine this principle with the *error bound formula* to place bounds on the uncertainty of the *nth-order Taylor polynomial approximation* of a definite integral.

The *binomial expansion* is given below,

$$(x + y)^N = \sum_{n=0}^{N} \binom{N}{n} x^n y^{N-n}$$

Euler's formula, true for all complex z, is given below,

$$e^{iz} = cos(z) + i\,sin(z)$$

Check Your Learning

1. Integrate by *Parts*: $\int x^2\, ln(x)\, dx$

(Rogawski 405)

2. $\int tan(x)\, sec^2(x)\, dx$

(Rogawski 413)

3. Integrate using a *trig substitution*: $\int \frac{dx}{x\sqrt{x^2+16}}$

(Rogawski 420)

4. Find the total *mass* of a 2-m rod whose linear *density* function is given by

$$p(x) = \frac{12}{x + 4}\ kg/m\ \ for\ \ 0 \le x \le 2$$

(Rogawski 375)

5. Find the *volume* of the *solid* obtained by *rotating* the region about the x-axis,

$$y = x^2 + 2\,, \qquad y = 10 - x^2$$

(Rogawski 382)

6. Use a *partial fractions decomposition* to find $\int \frac{25\, dx}{x(x^2+2x+5)^2}$

(Rogawski 434)

7. Determine whether $\int_0^{\infty} \frac{x\, dx}{(1+x^2)^2}$ *converges* or *diverges*.

(Rogawski 451)

8. What is the *work* done to lift a 10-m chain over the side of a building, if the chain has a constant *density* of 8 kg/m?

(Rogawski 397)

9. Compute the *surface area* of the *revolution* of $y = x^3$ about the x-axis over the interval [0, 2].

(Rogawski 479)

10. Solve the *initial value problem*: $t^2 \frac{dy}{dt} - t = 1 + y + ty, \ \ y(1) = 0$

(Rogawski 506)

11. Show that $\sum_{n=1}^{\infty} \frac{n}{10n+12}$ diverges.

(Rogawski 558)

12. Determine whether $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4n+9}$ *converges* or *diverges*.

(Rogawski 567)

13. Find the *interval of convergence* for the following *power series*:

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2^n n}$$

(Rogawski 589)

14. Compute the *Taylor polynomial* and find the maximum possible *error*:

$$f(x) = \frac{1}{\sqrt{x}}, \qquad center = 4; \qquad |f(4.3) - T_3(4.3)|$$

(Rogawski 600)

Things you may Struggle With

1. *Partial Fractions* is not integration. It is a method for simplifying an *integrand* to a point where it can be integrated. Once you perform a *partial fractions decomposition*, you must still integrate each of the resultant *fractions* individually in order to complete the original problem.

2. *L'Hoptial's Rule* only applies for limit expressions of the indeterminate form. If the limit expression is not in an indeterminate form, *L'Hopital's Rule* does not apply.

3. *Mass/Work*- when doing any physics-related integrations, do not get confused by the technical words. Just recognize what units things are in, what units the answer needs to be in, and accordingly, what kind of integration is required to get there. Because integration is generalized multiplication, integrating one variable with respect to another returns something in units that are the product of the units of the two variables being multiplied.

4. *Volume*- The key to volume integration problems is to figure out how to express the geometry of the solid in terms of a function. In other words, we are trying to express the area of the *cross section* as a function of the length along which we are integrating.

5. *Disk* and *Shell Methods*- One way to conceptualize the difference between the *Disk* and *Shell Methods* is to consider the geometry of the underlying *integrand*. The *Disk Method* takes the area of a circle for its *integrand* and integrates it along the axis of the *disk/washer*. The *Shell Method* takes the area of the (curved) rectangle as its *integrand* and integrates it from the center of the *shell* outward. See textbook for illustrations clarifying this distinction.

6. *Convergence Tests*- It can feel vary abstract and bewildering when one is tossed an infinite *series* and asked whether it *converges*. The key to getting started is to have the *tests*

memorized, to understand what the theorems are and what they mean. If you have the *tests* in mind and don't know which to choose, just pick one you feel good about. You might not pick the right one on the first try, but as you try a couple, you'll get quite good at it. Concretely, simply write down the beginning of the *test*, substitute in the specifics of the problem at hand, and do algebra until you get a limit you can evaluate.

7. *Sequence* and *Series* Symbology-

- n — an index (1, 2, 3, …)
- N — a particular point in the index
- $\{a_n\}$ — a *sequence*
- a_n — the n^{th} term of a *sequence*
- S — a (*convergent*) infinite sum (of a *series*)
- S_n — the n^{th} *partial sum* of a *series*
- ρ, L — limits

8. Integrating/Differentiating *series*- It is often necessary to integrate or differentiate an infinite *Taylor series term-by-term* to arrive at a solution. Of course, we do not actually integrate every term in the *series*, since that would take forever. Instead, we integrate the form of every term in the *series*, which will include n as an index. n looks like a variable, but it is important that when integrating *term-by-term*, we treat the index n as a constant, using x as our variable of integration.

9. Multiplying *series*- One can sometimes figure out what the product of two series will look like, just by multiplying the first few terms using FOIL. In this event, if one wants to get a glimpse at the terms of the product *series* up to, say *degree* 5, one should FOIL the two *series*, throwing out terms that result in a power of x greater than 5. Combining like terms algebraically, the result is the beginning of the *Taylor series expansion* of the product.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1. $\frac{x^3}{3}\left(ln(x) - \frac{1}{3}\right) + C$

2. $\frac{1}{2}\, tan^2 x + C$

3. $\frac{1}{4}\, ln\left|\frac{\sqrt{x^2+16} - 4}{x}\right| + C$

4. $12\, ln\left(\frac{3}{2}\right) \approx 4.87\ kg$

5. 256π

6. $ln|x| - \frac{1}{2} ln|x^2 + 2x + 5| + \frac{15 - 5x}{8(x^2+2x+5)} - \frac{13}{16} tan^{-1}\left(\frac{x+1}{2}\right) + C$

7. Converges (and is equal to 0.5)

8. 3920 J

9. $\frac{\pi}{27}(145^{3/2} - 1)$

10. $y = \frac{et}{e^{1/t}} - 1$

11. By the nth Term Divergence Test, $\lim_{n\to\infty} \frac{n}{10n+12} = \frac{1}{10} \neq 0$

12. Diverges

13. $[-\sqrt{2}, \sqrt{2}]$

14. $T_3(x) = \frac{1}{2} - \frac{1}{16}(x-4) + \frac{3}{256}(x-4)^2 - \frac{5}{2048}(x-4)^3$; $maximum\ error = \frac{35(0.3)^4}{65,536}$

References

Rogawski, Jon, et al. *Calculus: Early Transcendentals*. W.H. Freeman, Macmillan Learning, 2019.