Week 9
MTH-1322 – Calculus 2

Hello and Welcome to the weekly resources for MTH-1322 – Calculus 2!

This week is <u>Week 9 of class</u>, and typically in this week of the semester, your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Keywords: *Differential Equation, Particular Solution, General Solution, Initial Condition, Separation of Variables, Exponential Growth/Decay*

Topic of the Week: Differential Equations

Contents:

Highlight: 9.1 Introduction to Differential Equations

A *differential equation* is an equation relating a function to one or more of its derivatives. In algebra, we are often interested in solving for the value of a variable. In *differential equations*, we are often interested in solving for the value of a function or the form of a function. A function that makes a *differential equation* true is called a *particular solution* to the *differential equation*, and the form of all solutions to the *differential equation* is called the *general solution*.

Example: $\frac{dy}{dx} = y$.

One will perhaps recall that there is only one kind of function that is equal to its own derivative: the exponential function. Therefore, a *particular solution* to this *differential equation* is $y = e^x$, and the *general solution* is $y = ce^x$, where c is a constant.

A *differential equation* may also contain the variable of which y is a function. For example,

$$\frac{dy}{dx} = 1 - 6e^{2x}$$

To solve this equation, we multiply both sides by dx and then integrate to get the following,

$$y = x - 3e^{2x} + C$$

This is the *general solution* to the *differential equation* (Rogawski 499). A *particular solution* would choose a particular value for the constant C. Sometimes, one is given some extra bit of information called an *initial condition*. The *initial condition* narrows down the range of possible solutions to just one *particular solution*. Thus, any of a number of *particular solutions* in the form of the *general solution* might solve the *differential equation*, but only one *particular solution* will solve the *differential equation* and also satisfy the *initial condition*.

There is no easy algorithm for solving *differential equations*. Rather, there is a handbag of techniques for approaching *differential equations*, depending on how they can be classified. For example, some first-order (involving just the first derivative) *differential equations* can be manipulated into the form

$$\frac{dy}{dx} = f(x)g(y)$$

Such *differential equations* are called *separable*, and they have an easy method for solving. This method is called *separation of variables*, because it relies on the ability to factor y's derivative into a function of just x and a function of just y. The way forward from here is to multiply both sides by $\frac{dx}{g(y)}$ to get $\frac{dy}{g(y)} = f(x)dx$. All that is left to do from here is to integrate both sides, each with respect to its respective variable: $\int \frac{dx}{g(y)} = \int f(x)dx$. After performing this integration, one should be able to solve for y in terms of x, which is the solution to the *differential equation*.

For example, consider the following *differential equation*

$$\frac{dy}{dx} = 6xy^2$$

Notice that the derivative of y with respect to x is factorable into a function of just x (6x) and a function of just y (y^2). Applying the algorithm for *separation of variables*,

$$\frac{dy}{y^2} = 6x\,dx$$

$$\int \frac{dy}{y^2} = \int 6x\,dx$$

$$-\frac{1}{y} = 3x^2 + C$$

$$y = \frac{-1}{3x^2 + C}$$

This is the *general solution* to the *differential equation* (Rogawski 500).

Most *differential equations* are not solvable using *separation of variables*. However, for Calc 2 purposes, this is the only special method covered in the textbook. This means that unless your professor says otherwise, *separation of variables* is the main method you need to know for this class.

Differential equations have applications to many fields, especially physics. One common physical phenomenon describable with *differential equations* is *exponential growth and decay*. A function is said to *grow exponentially* when it is proportional to its own rate of change. In intuitive words: the more a thing grows, the faster it grows. The same applies for *exponential decay*, or shrinkage. This idea of a function's proportionality to its rate of change may be simply stated using *differential equations*:

$$\frac{dy}{dt} = ky$$

Solving this *differential equation* using *separation of variables*, one will obtain the following *general solution*:

$$y(t) = De^{kt}$$

The above formula is simply a more general case of the very first *differential equation* we looked at. This is because the only function that is equal to its own derivative is the exponential function e^x.

Consider a commonly used example of *exponential growth*. Let us say that a population of *E. coli* bacteria *grows exponentially*. If the initial population is 1000, and there are 1500 present after 1 hour, what function describes the population of *E. coli* bacteria after t hours?

Well, since the growth of the *E. coli* bacteria is *exponential*, if p(t) is the function describing the population after t hours, we know that p(t) must be in the form De^{kt}.

We plug in our information on the initial population: $1000 = p(0) = De^{k*0}$

And our information about the population at 1 hour: $1500 = p(1) = De^{k*1}$.

This yields 2 equations and two unknowns. We solve for the parameters D and k. After a little algebra, $D = 1000$ and $k = ln(1.5) \approx 0.405$ (Rogawski 503). Thus, the function requested is as follows:

$$p(t) = 1000e^{0.405t}$$

After recognizing the form of the *differential equation*, these *exponential growth* word problems really do not involve any more calculus. After getting the above equation, it is just a matter of remembering one's algebra to solve for special times, population sizes, and initial populations — whatever specifics the problem may demand.

Check Your Learning

1. Solve the *initial value* problem: $y\frac{dy}{dx} = xe^{-y^2}, \ \ y(0) = -2$ (Rogawski 506)

2. Solve the *initial value* problem: $t^2\frac{dy}{dt} - t = 1 + y + ty, \ \ y(0) = 0$ (Rogawski 506)

Things you may Struggle With

1. *Separation of variables*- These problems really test your algebra and order of operations more than anything. Be sure to remember the rules for manipulating exponentials (adding, multiplying, dividing, etc.) as you will have to do so frequently when getting these *differential equations* into a solvable form.

2. Word problems- Word problems requiring *differential equations* can be tricky the first time you see them. Draw pictures if necessary. Focus on giving important values mathematically symbolic names. Figure out what $\frac{dy}{dx}$ corresponds to. The intuition needed for these problems is similar to the intuition needed for related rates problems back in Calc 1.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1. $-\sqrt{ln(x^2 + e^4)}$

2. $y = \frac{et}{e^{1/t}} - 1$

References

Rogawski, Jon, et al. *Calculus: Early Transcendentals*. W.H. Freeman, Macmillan Learning, 2019.