Week 3
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is Week 3 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: Limits, Continuity, IVT, Derivatives

Highlight #1: Rates of Change

There are two rates of change: the average rate of change and the instantaneous rate of change. The **average rate of change** is a measure of how fast on average an object is moving across a period of time. Imagine driving from Waco to the airport.
The average rate of change from a point a to a point b, is calculated

$$y = \frac{f(b) - f(a)}{b - a}$$

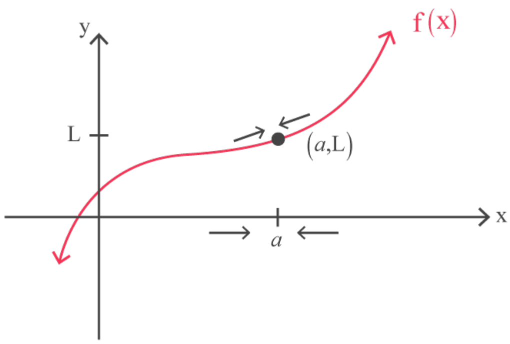
And would result in your average speed for the length of time you spent driving.

The **instantaneous rate of change** is the speed at one moment in particular during the trip.
The instantaneous rate of change at a point a is calculated by calculating the slope between the point a and a point really close to a.

$$y = \frac{f(a + 0.001) - f(a)}{(a + 0.001)\ - a}$$

Highlight #2: Limits

- The purpose of a limit is to describe the behavior of a function as it goes towards a specific value of X (or any other variable).
- The simplest way of finding a limit is to graph the function in question and see where it is going as X approaches the designated value from both sides.
- By plotting points closer and closer to the desired point, we can get a better and better idea of what the function is doing at that point



From the graph at the left we can tell that as x approaches a, f(x) approaches L.

We find the value L by:

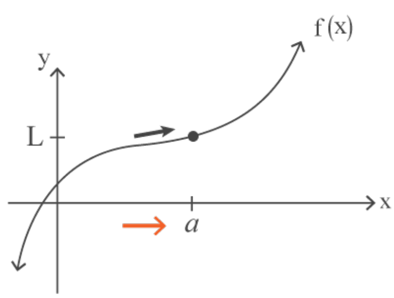
1) Looking at the graph at the point a
2) Plugging a into the function, and seeing what the function approaches

This graph shows **a two-sided limit**.

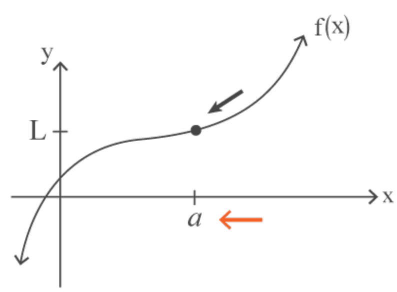
Let's look now at **one-sided limits.**
Sometimes the limit of a function may be different depending on which side you come from. A right-handed limit is the limit coming from the right side and going to the left, and the left-hand limit is the limit coming from the left side and going to the right.

We find left and right sided limits by:

1) Looking at the graph from the correct side
2) Plugging a number close to a (slightly larger if from the right, and slightly smaller if from the left) into the function, and seeing what the function approaches



$\lim_{x \to a^-} f(x) =$ left-sided limit



$\lim_{x \to a^+} f(x) =$ right-sided limit

You might realize now, that a two-sided limit can only exist if the limit from the left and the right equal each other

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Highlight #3: Limit Properties

There are several ways that we can manipulate a limit equation in order to make it easier to solve. Here are the most common rules:

Suppose that c is a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} f(x)$ exist. Then

Sum Law $\lim_{x \to a} \left(f(x) + g(x) \right) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

Difference Law $\lim_{x \to a} \left(f(x) - g(x) \right) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

Constant Multiple Law $\lim_{x \to a} \left(cf(x) \right) = c \lim_{x \to a} f(x)$

Product Law $\lim_{x \to a} \left(f(x) \bullet g(x) \right) = \lim_{x \to a} f(x) \bullet \lim_{x \to a} g(x)$

Quotient Law $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$

Power Law $\lim_{x \to a} \left(f(x) \right)^n = \left(\lim_{x \to a} f(x) \right)^n$

Root Law $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$, where n is a positive integer

Constant Law $\lim_{x \to a} c = c$

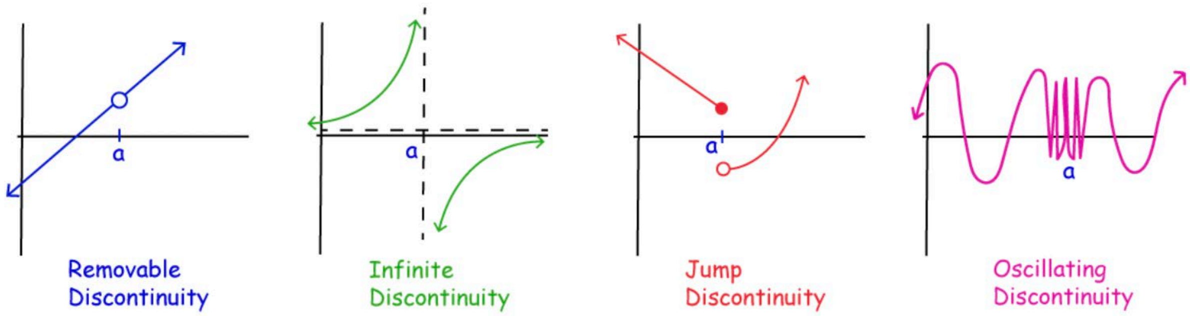
Direct Substitution Law $\lim_{x \to a} f(x) = f(a)$

Highlight #4: Limits and Continuity

A function is continuous at a point if the two-sided limit of the function exists at that point and equals the actual value of the function at that point. A function is continuous in general if it is continuous at every point.

There are several discrete types of discontinuity that can occur in a function:

- Removable discontinuities
- Infinite discontinuities
- Oscillating discontinuities
- Jump discontinuities



- Infinite discontinuities are vertical asymptotes.
- Removable discontinuities occur when the limit exists (The right and the left limit approach the same value, but the function is not defined at that point)
- Jump discontinuities occur when the left and right limit do not equal each other

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Highlight #5: Evaluating limits algebraically

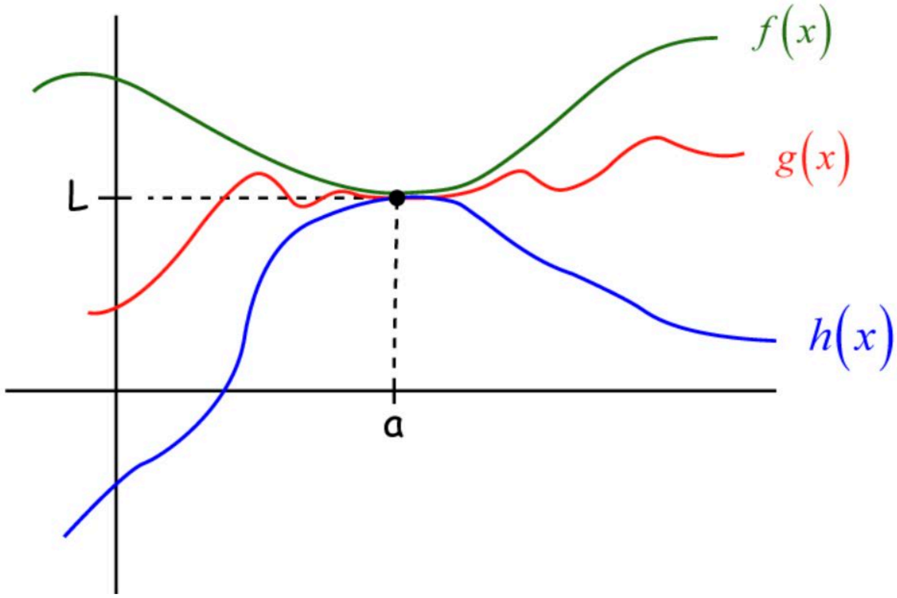
- The easiest way to **evaluate a limit** is to plug in the limit value into the function
- However, often that is not possible, as plugging in values results in indeterminate forms such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$ (which mathematically we still do not know how to evaluate)
- In such a case we have a few options:
  - Factoring
  - Using common denominators
  - Using conjugate pairs
  - Using trig identities to substitute

Highlight #6: The Squeeze Theorem and Trigonometry Limits

- The squeeze theorem is a theorem which allows us to evaluate certain tricky limits
- As we have seen before, certain functions oscillate at certain values, which makes it hard for us to evaluate
- The squeeze theorem allows us to evaluate the limits, by “squeezing” the more difficult function in between easier functions that we can evaluate at all values
- The Squeeze Theorem states:

If $h(x) \le g(x) \le f(x)$ when x is near a, except possibly at a,

and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$.



Where we are squeezing g(x) (the oscillating function) in between two easier and defined functions, h(x) and f(x)

- This video goes in depth about how to solve problems by using the squeeze theorem: https://www.youtube.com/watch?v=js45kis2ZoI&t=2s

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Highlight #7: Limits at infinity

In addition to using a limit to describe the behavior of a function on the part of the graph we can see, we can also use limits to describe the behavior of a function as it heads to infinity or negative infinity.
To calculate this limit we can look at asymptotes, as well as just the general behavior of the function.

Highlight #8: The Intermediate Value Theorem

The Intermediate Value Theorem states that for a continuous function, "for two numbers a and b in the domain of f, if $a < b$ and $f(a) \neq f(b)$, then the function f takes on every value between $f(a)$ and $f(b)$". The most common corollary of this theorem is that, if f(a) > 0 and f(b) < 0, at some point the graph must have crossed the x-axis, meaning there is a 0 on the interval from (a,b)

Highlight #9: Definition of the derivative

- The derivative is a function that describes the slope of a tangent line at a certain point on the graph
- Before we used to estimate it by finding two points that were as close as possible to the point we were interested in, but now we can estimate the derivative using limits
- You might remember from earlier, that the slope of the tangent line at one point is also the instantaneous rate of change, so by calculating the derivative we are also finding the instantaneous rate of change
- The derivative is marked by the symbol: $\frac{dy}{dx}$
- The limit definition of the derivative is as follows

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{x \to a} \frac{f(x) - f(a)}{x \to a}$$

- We use the first definition when we want the general formula of the derivative at any point, while we use the second formula when we are interested in the derivative at one specific point
- The following video gives a good example of how to use the limit definition to find the derivative:
  - https://www.youtube.com/watch?v=hxJFtwGSzsg&list=PLYjFOc4FIyim5VRQSDCC56kddVVRRS1ts&index=82

Check Your Learning

1. If it takes me an hour and a half to drive the 100 miles to Dallas, what was my average rate of change (average speed) while driving?

2. What are the right- and left-hand limits of $\lim_{x \to 0} \frac{1}{x}$?

3. Evaluate $\lim_{x \to \infty} (1 + \frac{1}{x})$

4. What is $\lim_{x \to -1} (\frac{x+1}{x^2-1})^2$?

5. What is the $\lim_{x \to \infty} \frac{70x^3+100x^2}{5x^4+35x}$?

6. What is the derivative of $3x^2 + 7x + 10$ at $\mathrm{x} = 4$?

Things Students Tend to Struggle With

- Indeterminate forms
  - If you get $\frac{0}{0}$ or $\frac{\infty}{\infty}$ when evaluating a limit, your next step should be to look for terms that can cancel in the numerator and denominator.
- Squeeze Theorem
  - The key to most squeeze theorem problems is finding the part of the original equation that is always bounded (usually $sin(x)$ and $cos(x)$) and then recreating your original function within the bounds of the inequality you identified.
- Limits at Infinity
  - The big takeaway from limits at positive or negative infinity is that only the largest terms in the numerator and denominator matter.
- Limit Definition of the Derivative
  - When using the $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ version of the limit definition of the derivative, the goal is to have an “h” in every term of the numerator, so that we can factor it out and cancel it with the one in the denominator.

- For $f\,'(x) = \lim_{h \to 0} \frac{f(x)-f(a)}{x-a}$ the same is true, except what you need to cancel is the "$x - a$" term.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1. 66.67 mph
2. The left-hand limit goes to negative infinity while the right-hand limit goes to positive infinity.
3. 1
4. ¼
5. 0
6. 31

All images are taken from Calcworkshop.com