Week 4
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is Week 4 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: Limits, Derivatives, Power rule, Product Rule, Quotient Rule

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Topic of the week: Derivatives

- The derivative is a function that describes the slope of a tangent line at a certain point on the graph
- Before we used to estimate it by finding two points that were as close as possible to the point we were interested in, but now we can estimate the derivative using limits
- Let's work through an example together, if you have any more questions, please refer to this video
    - https://www.youtube.com/watch?v=hxJFtwGSzsg&list=PLYjFOc4FIyim5VRQSDCC56kddVVRRS1ts&index=82

- Ex. Find the derivative of f(x) = $\sqrt{x^2 - 4}$ at the point x = 5
    - From the limit definition of the derivative we know that
        - $\frac{dy}{dx} = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h} = \lim_{h \to \infty} \frac{\sqrt{(x+h)^2 - 1} - \sqrt{x^2 - 1}}{h}$
    - When using this version of the Limit Definition of the Derivative, our first goal is to manipulate the equation until we can pull an "h" out of the entirety of the numerator of the equation. For this equation, that means simplifying the numerator by multiplying it by its Conjugate.

- $$f'(x) = \lim_{h->0} \frac{\sqrt{(x+h)^2-1} - \sqrt{x^2-1}}{h}$$
  $$= \lim_{h->0} \frac{\sqrt{(x+h)^2-1} - \sqrt{x^2-1}}{h} * \frac{\sqrt{(x+h)^2-1} + \sqrt{x^2-1}}{\sqrt{(x+h)^2-1} + \sqrt{x^2-1}}$$
- Let's simplify
  - $$f'(x) = \lim_{h->0} \frac{x^2+2xh+h^2-1-x^2+1}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})} = \lim_{h->0} \frac{2xh+h^2}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})}$$
- Now that we have an "h" that we can pull out of the numerator we can cancel our "h" in the denominator
  - $$f'(x) = \lim_{h->0} \frac{h(2x+h)}{(h)(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})} = \lim_{h->0} \frac{(2x+h)}{(\sqrt{(x+h)^2-1} + \sqrt{x^2-1})}$$
- Now that we have canceled the "h" that was in our denominator, we should be able to evaluate the limit by plugging in 0 for all our remaining "h's"
  - $$f'(x) = \frac{(2x+0)}{(\sqrt{(x+0)^2-1} + \sqrt{x^2-1})} = \frac{2x}{2\sqrt{x^2-1}} = \frac{x}{\sqrt{x^2-1}}$$
- Now all we have to do is plug in x=5 to get our answer
  - $$f'(5) = \frac{5}{\sqrt{24}}$$

Highlight #1: The Derivative as a Function

- So far, we always wanted to find the derivative at a specific point, so we always had a value to plug in.
- However, if we are simply interested in the slope at any point we don't have to plug in a point, and we can keep the derivative as a function of x
- The method is the same, we are only missing the last step where we plug in a value for x

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Highlight #3: Power rule

Now that you have done the limit definition you will find the new way to find the derivative a lot easier to compute. It is called the power rule. It is a way to accurately calculate the derivative of a function without the hassle of using the limit definition.
Mastering the power rule is essential to succeeding in this class.

The power rule states:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

In order to apply the power rule, we need to follow these steps:

1. Move the exponent down in front of the variable
2. Multiply it by any coefficient that was in front of the variable
3. Decrease the exponent by 1

The following video goes over the power rule:

- https://www.youtube.com/watch?v=EyN92I3jk1k

Other significant rules are:

- the sum rule, which states that the derivative of the sum is the sum of the derivatives
  $$(f + g)' = f' + g'$$
- the difference rule, which states that the derivative of a difference is the difference of the derivatives
  $$(f - g)' = f' - g'$$
- the constant multiple rule, which allows us to multiply the derivative by the coefficient
  $$(cf)' = cf'$$

Highlight #2: Product and Quotient Rules

- Often we want to find a derivative of a product of two functions or the quotient of two functions
- Take for example h(x) 3x * sin(x), it would be very complicated to find the derivative of this function using the limit definition
    - However, if we can split h(x) into a product of f(x) = 3x and g(x) = sin(x), we can use the product rule to find the derivative
    - The product rule is as follows
      $$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$
    - Where f(x) and g(x) are functions we identify in the original function
    - To remember the product rule, just repeat "first times the derivative of the second + second times the derivative of the first"

- We also have a rule that allows us to take the derivative of a quotient of functions
- Take for example h(x) = $\frac{x+5}{x^2+7}$, finding the derivative using the limit definition would be tedious, but if we let f(x) = x + 5, and g(x) = $x^2 + 7$, we can use the quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

   - To remember the quotient rule, I memorized the mnemonic "low d high – high d low all over the square of the low"
   - Remembering f and g might get confusing especially as exercises start using other letters, so identify them by thinking of them as the top function and the bottom function
- The following video does a very good job showing examples of the product and quotient rule
   - https://www.youtube.com/watch?v=kxqmLULgiTI

Check your Learning:

1. What is the derivative of $3x^2$?

2. What is the derivative f(x) = $\frac{x^3}{\ln(x)}$?

Things you might struggle with:

- Limit Definition of the Derivative
  - When using the $f\ '(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ version of the limit definition of the derivative, the goal is to have an "h" in every term of the numerator, so that we can factor it out and cancel it with the one in the denominator.

- Product Rule: It is often easy to forget to apply the Product Rule when taking derivatives – especially on longer problems that have many layers. In general, the best way to avoid these kinds of silly mistakes is to take your time and be methodical while working through problems.

- Quotient Rule: All of the same issues with the Product Rule also apply for the Quotient Rule, but making everything more difficult is the fact that the formula for the Quotient rule is much more complicated. In general, I find it very helpful to try and memorize the Quotient Rule by using a jingle or some other memorization technique.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Solutions to Check your Learning

1. 6x
2. f'(x) = $\frac{\ln(x)*3x^2 - x^2}{(lnx)^2}$

All images are taken from Calcworkshop.com