Week 6
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is Week 6 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: Implicit Differentiation, Inverse Trig Functions, log and ln Derivatives

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Topic of the Week: Implicit Differentiation

So far, all of our functions have had all the "y's" on one side and all of the "x's" on the other, for example, $y = 5x + 6$, which meant that taking the derivative was a pretty straightforward process.

What happens when we want to take the derivative of a function where the variables appear on both sides? For example, in a function such as $y^2x + x^2y = xy - 2$?
The method that allows us to find the derivative of such a function is called implicit differentiation, and it has 3 steps to it:

1. Take the derivative of all the variables that appear in the equation
2. When you take the derivative of the variable y, remember to multiply it by $\frac{dy}{dx}$
3. Solve for $\frac{dy}{dx}$

Here is a worked through example

$x^2 + y^2 = 36$	*Implicit equation (x and y intermixed)*
$2x + 2y\left(\frac{dy}{dx}\right) = 0$	*Take the derivative of every variable, but when we take the derivative of "y" we multiply by* $\frac{dy}{dx}$
$2y\left(\frac{dy}{dx}\right) = -2x$	*Substract 2x from both sides of the equation*
$\frac{dy}{dx} = \frac{-2x}{2y}$	*Divide both sides of the equation by 2y*
$\frac{dy}{dx} = \frac{-x}{y}$	*Simplify*

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The following video goes more in depth about the process of implicit differentiation:
https://www.youtube.com/watch?v=M-fdkDEhuZc

Highlight #1: Inverse Trig

The inverse trig derivatives look very difficult, and the proof for how they are derived is not taught until Calculus 2. Although it may be hard, I recommend memorizing them, as it will make exercises and problems much easier (especially when dealing with combinations such as power rule or chain rule)

$$\frac{d}{dx}\left[\sin^{-1}\left(f(x)\right)\right]=\frac{f'(x)}{\sqrt{1-\left(f(x)\right)^2}}$$

$$\frac{d}{dx}\left[\cos^{-1}\left(f(x)\right)\right]=\frac{-f'(x)}{\sqrt{1-\left(f(x)\right)^2}}$$

$$\frac{d}{dx}\left[\tan^{-1}\left(f(x)\right)\right]=\frac{f'(x)}{1+\left(f(x)\right)^2}$$

$$\frac{d}{dx}\left[\cot^{-1}\left(f(x)\right)\right]=\frac{-f'(x)}{1+\left(f(x)\right)^2}$$

$$\frac{d}{dx}\left[\sec^{-1}\left(f(x)\right)\right]=\frac{f'(x)}{\left|f(x)\right|\sqrt{\left(f(x)\right)^2-1}}$$

$$\frac{d}{dx}\left[\csc^{-1}\left(f(x)\right)\right]=\frac{-f'(x)}{\left|f(x)\right|\sqrt{\left(f(x)\right)^2-1}}$$

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Highlight #3: Derivative of exponentials, logs, and ln

Taking the derivatives of exponentials, logs, and lns have certain steps you must follow that are not the usual steps we have gotten used to.

Here are the steps to differentiating exponential functions:

1. Rewrite the original function
2. Multiply the original function by the natural log of the base
3. Multiply by the derivative of the exponent

$$\frac{d}{dx}\left[a^{f(x)}\right]=\underbrace{a^{f(x)}}_{rewrite}\cdot\underbrace{\ln a}_{\substack{natural\ log\\ of\ base}}\cdot\underbrace{f'(x)}_{\substack{derivative\\ of\ exponent}}$$

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Here are the steps to differentiating logarithmic functions:

1. Take the derivative of the function
2. Divide by the natural log of the base and the original function

$$\frac{d}{dx}\left[\log_a f(x)\right] = \frac{\overbrace{f'(x)}^{\text{derivative of function}}}{\underbrace{\ln a}_{\text{natural log of base}} \cdot \underbrace{f(x)}_{\text{rewrite}}}$$

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Notice! Ln is simply a log-based e, so the derivative of

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)\ln(e)} = \frac{f'(x)}{f(x)}$$

I would recommend memorizing them if you can!

Check your Learning

1. Find the derivative of $y^3 + e^x = 27$ using implicit differentiation

2. Find the derivative of $\cos^{-1}(x^2)$

3. Find the derivative of 7^x

Things you might struggle with:

- It is easy to confuse the power rule with implicit differentiation!
  - Remember that you still follow the power rule process, you just need to add a $\frac{dy}{dx}$ whenever you see a y
- Memorize the inverse trig function formulas and when to use them.
  - They can be tricky, so I recommend finding little tricks to help you differentiate between them.

- For example the inverse tangent derivatives have addition in the denominator with no radical while the sin and cosine do not.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check your Learning

1. $\frac{dy}{dx} = -\frac{e^x}{3y^2}$
2. $\frac{2x}{\sqrt{1-(x^2)^2}}$
3. $ln(7) * 7^x$

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