Week 7
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is Week 7 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: Related Rates, Linear Approximation, Critical Points

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Topic of the Week: Related Rates

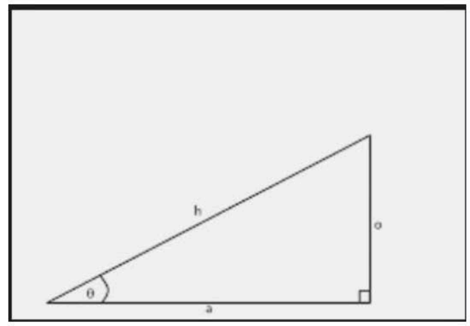
Related rates are one of the most difficult things in a Calculus 1 course, so be patient with yourself.

We actually use and think about related rates all the time! If you're a college student you probably have a Brita somewhere in your room, and you might have asked yourself how long you can fill the Brita while the filter works before the Brita overflows. That's a related rate problem! As you have two rates interacting with each other.

These are the steps to solving related rates problems:

1. Read the problem carefully! It will sound confusing so take note of every number value you see and **identify what you are looking for**
2. Draw a diagram, **identify and label what you know** (often it will be a triangle, or a rectangle)
3. **Find an equation** that relates a piece of information you already know to the unknown piece you are solving for (it might be an area formula, a trig identity, or a theorem)
4. **Differentiate the equation** with respect to time (as we are talking about rates, time will always be mentioned)
5. **Substitute all values** into the derivative and solve

Ex. 1) A ladder is against a building and slowly sliding down. Let h(t) be the height of the top of the ladder to the ground at time t, let x(t) be the distance between the base of the building and the base of the ladder at time t, the ladder is 5 meters long, x(0)=1.5, and the speed at which the base of the ladder is moving away from the building is .8 m/s. Find the speed at which the height of the ladder (h(x)) is changing at t=1.



Step 1. First ask yourself what are you looking for?

$$\frac{d}{dt}h(t) \; at \; t = 1$$

Step 2. Ask yourself what do you know?

x(0)=1.5 $\quad m = \frac{dx}{dt} \quad \frac{dx}{dt} = .8$

$x(t) = mt + x(0)$

Step 3. Using Pythagorean Theorem our equation is

$$h(t)^2 + x(t)^2 = 5^2$$

Step 4. We use implicit differentiation to find how how each part of the equation changes over time

$$\frac{d}{dt}h^2 + \frac{d}{dt}x^2 = \frac{d}{dt}5^2$$
$$2h\frac{dh}{dt} + 2x\frac{dx}{dt} = 0$$
$$\frac{dh}{dt} = \frac{-2x}{2h} * \frac{dx}{dt}$$

Step 5.From here, find x(1), h(1), and plug them and dx/dt = .8 into the formula we just created to find the answer.

Answer: $\frac{dh}{dt} = -.41$

The following video goes over related rates:
https://www.youtube.com/watch?v=J_mum2BNcS4

Highlight #1: Linear Approximation

The process of linearization allows us to estimate very specific values without having to use a calculator. Linearization relies on the fact that if you zoom in again and again on any part of a graph, the graph resembles a line. Therefore, we can calculate the tangent line to approximate values that are near to a particular point we identified.

The equation of the tangent line looks like this:

L(x) = f(a) + f'(a)(x-a)

Which may look new, but is really just a different way of writing a line

$$y - y_1 = m(x - x_1)$$
$$y = y_1 + m(x - x_1), \text{ and if } (x_1, y_1) = (a, f(a)) \text{ and } m = f'(a) \text{ then}$$
$$y = f(a) + f'(a)(x - a)$$

Here are the steps to Linear Approximation:

1. Find the point we want to zoom in on (the prompt will usually hint at it)
2. Calculate the slope at that point using derivatives
3. Write the equation using point slope form
4. Plug in the nearby point in the equation and solve

Here is a worked example step-by-step:

Find the linearization of the function $f(x) = \sqrt[3]{x}$ at $a = -8$ and use it to approximate the number -8.1

1. Plug in $a = -8$ for x and solve for y to find ordered pair	$f(a) = \sqrt[3]{x}$ $f(-8) = \sqrt[3]{-8} = -2$ $(-8, -2)$
2. Take the derivative to find the slope of the tangent line	$f(x) = \sqrt[3]{x} = x^{1/3}$ $\frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$
3. Plug in the ordered pair from step 1 and solve for the slope: $\frac{dy}{dx}\Big\vert_{x=-8, y=-2}$	$\frac{dy}{dx} = \frac{1}{3(-8)^{2/3}}$ $m = \frac{1}{3(-2^{3})^{2/3}} = \frac{1}{3(-2)^{2}}$ $m = \frac{1}{12}$
4. The Linearization is found by substituting the ordered pair and slope found in step 3 into Point-Slope Form	$y - y_1 = m(x \quad x_1)$ $y - (-2) = \frac{1}{12}(x - (-8))$ $y + 2 = \frac{1}{12}(x + 8)$ $y = \frac{1}{12}x - \frac{4}{3}$
5. Find the Linear Approximation of the number -8.1, plug it into the equation of the tangent line	$y = \frac{1}{12}x - \frac{4}{3}$ $y = \frac{1}{12}(-8.1) - \frac{4}{3}$ $y = -2.008$

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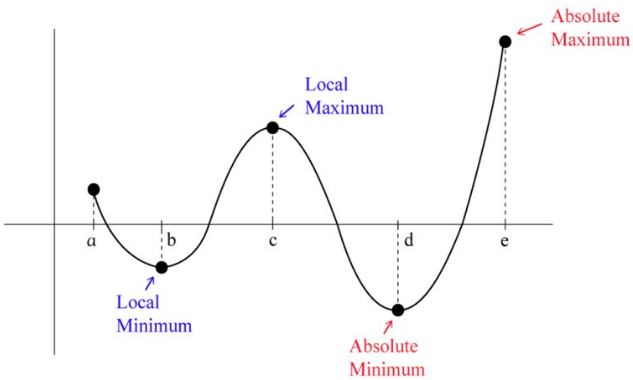
The error and percentage error for a linear approximation are defined as:

- Error = $|\Delta f - f'(a)\,\Delta x|$
- % error = $\left|\frac{errror}{actual\ value}\right| * 100\%$

Highlight #2: Critical Points

Any continuous function f(x), that is defined on an interval [a,b], has an extremum (or critical point) in that interval.
You might ask yourself what an extremum is, and the mathematical definition of an extrema is "any point that is either the highest or the lowest in the interval".
There can be absolute extrema, meaning it is the highest/lowest point on the entire domain, or a local extremum, which is the highest/lowest point on a particular interval.
Extremas occur whenever the derivative is equal to 0. If you look at the following graph (or any graph), you will notice that whenever there is a maximum or minimum, the tangent line at that point is horizontal (meaning the slope of the tangent line is 0)



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To find an extrema we must:

1. Find the derivative
2. Set the derivative equal to 0
3. Solve the equation for x
4. Ensure that x is in the interval/domain of the function

To determine whether the extrema we found is a min or a max we must look at the slopes of the points surrounding it:
-if the slope to the left is positive, and the slope to the right is negative, then the point is a maximum
-if the slope to the left is negative, and the slope to the right is positive, then the point is a minimum

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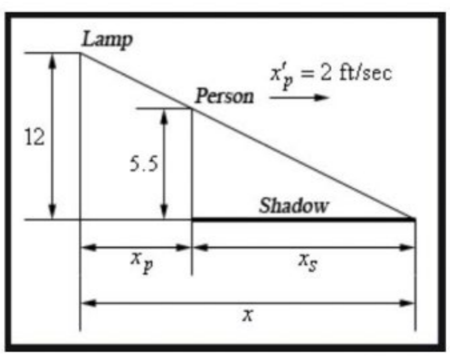
Check your Learning

1. A 5.5ft person is walking away from a lamp at a speed of 2ft/second. A 12ft lamp is behind him casting a shadow in front of him. Let x(p) be the distance from the lamp to the person at time t, let x(s) be the distance of the shadow at time t.
   How fast is the tip of the shadow moving away from the person?



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2. Estimate x^3 from the point x = 2 to the point x = 2.03.

3. What are the critical points of f(x) = $x^2 + 10\,x + 25$? Are they max or mins?

Things you might struggle with:

- Related rates can be difficult, especially since you are coming up with your own equation
    - Always start a problem by drawing a picture
- Linearization looks difficult, but it us just a way of writing the equation of a line, you already have all the needed skills!

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1) $x_s = 1.692$
2) L(2.03) = 8 + 12(2.03) = 8.36
3) x = -5(minimum)