Week 8
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is <u>Week 8 of class</u>, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: optimization, mean value theorem, first and second derivative

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Topic of the Week: Optimization

Optimization is the process of finding maximum and minimum values given a constraint.

You might see problems asking you to maximize the area of a certain enclosure knowing the length of the fence, minimizing the cost of a product, or travel time.

There are 8 easy steps to optimization:

1. "Translate" the problem by assigning variables and sketch the situation described in the problem
2. Find the constraint equation
3. Find an equation that includes what you are trying to evaluate (secondary equation)
4. Solve the constraint equation for a variable, and plug the obtained value in the secondary equation
5. Simplify the secondary equation
6. Differentiate the secondary equation
7. Set the derivative equal to 0 and find the critical points
8. Verify the critical points are maximums/minimums depending on what the problem is asking

The following video covers optimization: //www.youtube.com/watch?v=X6fEBmyL-18

Highlight #1: Mean Value Theorem

In this chapter, we will learn the Mean Value Theorem or MVT.
The MVT states that, across an interval of a continuous function on [a,,b] and differentiable function on (a,,b)at some point between the endpoints of the interval there will be a point c, such that f'(c) = $\frac{f(b)-f(a)}{b-a}$

Simply put: if the function is continuous and a differentiable function, there must be a point on the line where the instantaneous rate of change equals the average rate of change.

Ex. 1 Find a "c" satisfying the conclusion of MVT for $f(x) = x^3$ on the interval [0,5].

Step 1. Find average rate of change of the function.

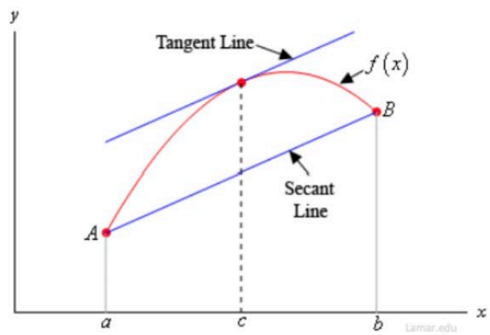
$$\frac{f(b)-f(a)}{b-a} = \frac{125-0}{5-0} = 25$$

Step 2. Take the derivative of the function.

$$\frac{d}{dx}x^3 = 3x^2$$

Step 3. Set the derivative equal to the average rate of change and solve for x.

$$1)\ 3x^2 = 25 \quad 2)\ x^2 = \frac{25}{3} \quad 3)\ x = \sqrt{\frac{25}{3}} \approx 2.88$$



The following video goes over the Mean Value Theorem:
https://www.youtube.com/watch?v=IQ9OVpSz-iE

Highlight #2: Derivative Test

The first and second derivative test can give us a lot of information regarding the graph of the function, in fact, we will be able to graph a function simply by calculating and evaluating the first and second derivative.

- Maximums and minimums:
  - Represented by x-axis intercepts (or values of f'(x) = 0)
- Points of inflection
  - Represented by the zeroes of the second derivative
- Increasing and decreasing portions of the function

- Discerned from the sign of the first derivative
- f'(x) is positive means the graph is increasing, f'(x) is negative means the graph is decreasing

- Concave up (graph looks like a smiley face) and concave down (graph looks like a frown)
  - When the graph is concave up, the values of the second derivative are positive
  - When the graph is concave down, the values of the second derivative are negative

Ex. 1 Where is the graph of $x^5 - 5x^4$ concave up and concave down?

Step 1. Take the second derivative of the function.

$$f'(x) = 5x^4 - 20x^3$$

$$f''(x) = 20x^3 - 60x^2$$

Step 2. Set the second derivative equal to 0.

$$f''(x) = 20x^2(x-3)$$

$$20x^2 = 0 \rightarrow x = 0$$
$$x - 3 = 0 \rightarrow x = 3$$

Step 3. Plug in a Value that is greater and less than the critical point.

$-1 < 3,\ 20(-1)^2(-1-3) = -80$
$5 > 3,\ 20(5)^2(5-3) = 1000$

Step 4. Write the concavity based on the second derivative test.
It is concave down from negative infinity to 3 and concave up from 3 to positive infinity.

Check Your Learning

1. Minimize the surface area of an open-topped box with a volume of $100 meters^2$, given that the base of a box is a square. What is the height of the box?

2. Find a "c" satisfying the conclusion of MVT for f(x) = $3x^3$ on the interval [0,3]

3. Find the critical points of the function f(x) = $x^3 - 3x^2$ on the interval [-5,5] and determine if they are local maximums or minimums using the first derivative test

Things you might struggle with:

- Optimization: We have learned to take the higher order derivatives of functions as well as finding the local minimums and maximums. So, the only unfamiliar step in optimization is constructing the objective and constraint equation at the beginning of a optimization problem. I highly recommend drawing diagrams to help understand the problem you are given. Often the math faculty only asks a couple different optimization problems on the exam so exposing yourself to the select few kinds of problems will help you be able to easily recognize and address the questions given on the exam.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning:

1. h = 2.92
2. $x = \frac{\sqrt{15}}{3} = 1.29$
3. The two points are x = 0 (a local max) and x = 2(a local min)