Week 9
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is Week 9 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: Riemann Sums, Definite Integrals, Indefinite Integrals

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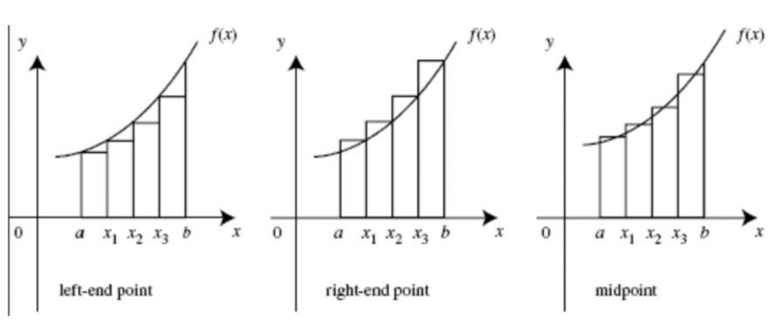
Topic of the Week: Riemann Sums

Often, it is of our interest to calculate the area below a curve. While it may seem daunting, especially if the area under the curve does not look like a recognizable shape, we have a new tool that can help us do it. This tool is called Reimann Sums.
A Riemann Sum approximates the area underneath a curve through the use of rectangles-the width being the interval of x, the height being f(x). These rectangles take on the height of the graph at some point on the interval of the rectangle – usually the right, left, or middle of said curve. By summing up the areas of all the rectangles you created, you can find an approximation area of the area under this curve.



Mathequality.wordpress.com

$$\sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

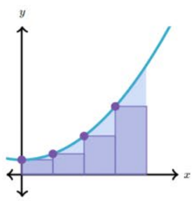
Where
n = number of rectangles
f(x_k^*) = height of each rectangle
Δx_k = width of each rectangle

These are the steps to evaluating Riemann sums:

1) Draw a picture to visualize the curve
2) Evaluate the change in x, Δx_k
3) Use f(x) to find the height of the rectangle, and sum up the areas

Ex. 1) What is the Riemann approximation of the area under $f(x) = x^2$ on the interval [1,4] using a left-hand approximation and three rectangles of equal Δx?
Is this an over or under estimation of the actual area under the curve?

Step. 1) Draw picture to help visualize.



Step. 2) Solve for the change in x.

Four can be evenly split into 3, 1 unit wide rectangles.

Step 3.) Use the equation to find the height of the rectangle then add the areas together.

A: $L3 = (1 * 1) + (4 * 1) + (9 * 1) = 14.$

The following video goes over Riemann sums:
https://www.youtube.com/watch?v=gK0K1XptyNA

Highlight #1: Definite Integrals

Riemann sums can get pretty painful if we have more than 5 rectangles. This is why we will add another tool to our toolbox, called definite integrals, which allow us to measure the area under the curve within a certain interval. The way Riemann sums work is by making the width of the rectangles get closer and closer to 0 (therefore making the number of rectangles go to infinity!)

These are the rules when taking definite integrals:

- $\int_a^b c dx = c(b-a)$
- $\int_a^b x^n dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^a f(x)dx = 0$

- If M≤ f(x) ≤ m on the interval (a,b), then $\int_a^b m dx \leq \int_a^b f(x)dx \leq \int_a^b M dx$

- This rule also works with functions!

Ex. 2) $f(x) = \int_2^4 x^2 dx$

Step 1.) Take the anti-derivative.

$$x^n = \frac{x^{n+1}}{n+1} + C = \frac{x^3}{3}$$

Step2.) Use the fundamental theorem of calculus to solve the definite integral.

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \qquad \frac{(4)^3}{3} - \frac{(2)^3}{3} = \frac{56}{3}$$

The following video goes over taking definite integrals:
https://www.youtube.com/watch?v=D9o6Gbg77OM

Highlight #2: Indefinite Integrals

In chapter 5.3 we are learning about indefinite integrals.
The only difference between definite and indefinite integrals is the presence (or lack thereof) of endpoints of the integral. Since the indefinite integral doesn't have bounds, two things will be different from calculating a definite integral.

1. We will get an equation for our answer, rather than a value
2. We will need to add a "+c" to all of our equations.

This section also introduces new integral rules

- $\int 0dx = c$
- $\int \sin(kx)dx = -\frac{1}{k}\cos(kx) + c$
- $\int \cos(kx)dx = \frac{1}{k}\sin(kx) + c$
- $\int e^{kx}dx = \frac{1}{k}e^{kx} + c$
- $\int \frac{1}{x}dx = ln|x| + c$
- $\int cf(x)dx = c\int f(x)dx$
- $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$

Ex. 3) $f(x) = \int (x^2 + \frac{1}{x})dx$

Step 1.) Take the anti-derivative.

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

$$\frac{x^3}{3} + \ln(x)$$

Step 2.) ADD "+c" to the solution.

$$\frac{x^3}{3} + \ln(x) + c$$

The following Baylor tutoring video goes over indefinite integrals:
https://www.youtube.com/watch?v=pv1MthFqngM

Check your Learning

1) $Calculate\ Middle - Point\ Reimann\ Sum\ of\ \int_0^3 x^3\ \ N = 6$

2) $\int_2^5 7x^2 + 4x + 9$

3) $\int 5x^5 + e^x$

Things you might struggle with:

- Area Under the Curve
  - The calculating part for Riemann sums is not difficult, but the concept behind Riemann sums is fundamental to understanding integral calculus.
  - To find the area under the curve, we are essentially adding up the area of rectangles under curve. The larger the width of the rectangle the less accurate the area is (look at the definite integral section for illustration).
  - So when calculating Riemann sums you are deciding the width, or change in x, of the rectangles under the curve. When we are taking the integral (definite or indefinite), we are using rectangles with an infinitely small width and change in x, to calculate the exact area under the curve.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Solutions to Check your Learning

1. **19.97**
2. **342**
3. $\boldsymbol{e^x + \frac{7}{6}x^6}$