Week 13

Physics 1409/1430 – Physics 2

Hello and Welcome to the weekly resources for PHY 1409/1430 – Physics 2!

This week is Week 13 of classes, and typically in this week of the semester, your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout the semester.

We also invite you to take a look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30-minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours, M-Th 9am-8pm on class days, at 254-710-4135.

Keywords: Wave Function, Heisenberg Uncertainty Principle, Quantum Numbers, Pauli Exclusion Principle

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The Wavefunction:

Earlier, we inspected the foundational concepts and discoveries that led to the birth of quantum mechanics. We learned that energy is quantized. We have also learned the there is wave-particle duality to matter as well. It changed how we look at the world. The impact of the wave-particle duality is seen mainly in a very small scale, so the laws of physics that you have studied so far are still valid and close enough to not require any changes.

These quantum phenomena led to the development of a new theory to make sense of the behavior, which we know today as quantum mechanics. It successfully unifies the wave-particle duality and the spectra emitted by atoms.
When we consider light, we use the components of waves like wavelength, frequency, and amplitude to describe the different kinds of electromagnetic waves. As matter has a wave nature, we can do the same for matter waves. The waveform that we see in quantum mechanics when we look at matter is called the wavefunction ($\Psi$). The wavefunction represents the displacement of the wave as a function of time and position. So, for a given time and position, the wavefunction describes the behavior of the matter wave.

Due to this wave nature of matter and the small size of the particles in consideration, quantum mechanics adopts a statistical interpretation to understand how a particle is behaving. The square of the wavefunction represents the probability distribution over space and time. A common use is looking at an electron in an atom. The square of the wavefunction will tell us the probability of finding an electron at a given position at a particular time.

**Heisenberg Uncertainty Principle:**

As you are now aware from all the physics labs you have done so far, there is always uncertainty in the measurements you make with your instruments. Some instruments are incredibly precise but there is always some degree of uncertainty. In quantum mechanics, we see that nature imposes an uncertainty on everything, it is not only inherent in the process of measurement. Uncertainty comes from the wave-particle duality of matter and the interaction between the instrument of measurement and the system. But in quantum mechanics, there is a limit of uncertainty in everything. This uncertainty puts a limit on the accuracy of the position and momentum of a particle (electron in most cases here). This uncertainty is called the Heisenberg Uncertainty principle. It provides the lowest uncertainty limit.

$$\langle \Delta x \rangle \langle \Delta p_x \rangle \geq \frac{\hbar}{2\pi}$$

It tells us that we can never know the precise position and momentum of a particle at any given time. It also tells us that the more accurately we know the position or momentum of a particle, the precision for the other variable decreases.

The uncertainty principle not only relates momentum and position, but also energy and time.

$$\langle \Delta E \rangle \langle \Delta t \rangle \geq \frac{\hbar}{2\pi}$$
The $h$ is the plank’s constant. We use another plank’s constant value here as well

$$h = \frac{\hbar}{2\pi} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}$$

**Quantum Mechanical View of the Atom:**

In the realm of quantum mechanics, everything is mired by indeterminacy. We do not know the conditions or characteristics of an atom accurately. In the classical view, the macroscopic scale, we can know the velocity, momentum, and physical characteristics of object. But for atoms in the quantum realm, we are not so sure. So, we must adopt a new way to look at systems. We judge the possibilities for the state of particle based on what is allowed by the energy. So, we can find the probability of a particle being at a particular point in space. This probabilistic view of quantum mechanics is referred to as the Copenhagen interpretation.

So, taking the probabilistic perspective to look at the atom, lets us see how it affects their characteristics. Based on the wavefunction, we can build the probability distributions, which show the probability of a particle being in a point in space. An easy example to consider is the hydrogen atom.

As we know that we cannot accurately know the position of the electron, we see the electron as a cloud around the nucleus, which shows the possible space in which the electron is present (figure on the right).

So how do we now describe the electron? We use the quantum numbers to describe the basic energy levels and the state of the electron. The following tells us the basic energy level of the electron based on the shell it is present in.

$$E_n = -\frac{13.6 \text{ eV}}{n^2}, \quad n = 1, 2, 3, \cdots,$$
In quantum mechanics, there are 4 different quantum numbers:

1. The principle quantum number \( n \), which determines the total energy state of the electron in the hydrogen atom.

2. The orbital number \( l \), which is related to the angular momentum of the electron, which takes integer values from 0 to \((n-1)\). The magnitude of angular momentum \( L \) is

\[
L = \sqrt{l(l + 1)} \hbar
\]

3. The magnetic quantum number \( m_l \), which relates to the direction of the angular momentum and takes the value \(-l\) to \(l\).

4. The spin quantum number \( m_s \), which can have only two values for an electron, \(-1/2\) or \(1/2\).

Based on these quantum numbers, we determine the possible electron clouds, which may look familiar to you if you have taken chemistry. The probability distributions form orbitals.

There are limitations in what is allowed as far as movements of electrons are concerned. An electron can only move to adjacent orbitals and cannot jump across multiple orbitals. These transitions are forbidden.

Things are more complicated with atoms bigger than hydrogen with multiple electrons. When looking at multiple electrons, another principle comes into play, the Pauli Exclusion Principle. It states that no two electrons can occupy the same quantum state. So no two electrons can have the same exact quantum numbers that we stated above.
CHECK YOUR LEARNING

1. According to the uncertainty principle,
   a. There is always an uncertainty in a measurement of the position of a particle
   b. There is always an uncertainty in the measurement of the momentum of a particle
   c. There is always an uncertainty in a simultaneous measurement of both the position and momentum of a particle.
   d. All of the above
2. An electron remains in an excited state within the atom for $10^{-10}$ s. What is the minimum uncertainty in the energy of the state?
3. How many electrons can be in the n=3 shell? What are the values of l, m_l and m_s possible?

THINGS YOU MAY STRUGGLE WITH

1. The concepts discussed in quantum mechanics can be very confusing and weird as it is not how we are used to thinking. Remember that in this realm, we know nothing for certain until we measure the system. Even with measurement, there will be uncertainty.
2. The electron cloud and probability can be confusing. Remember that the cloud describes the possible space in which the electron can exist around the atom and that this space is derived from the probability distribution of the electron position. It is also derived from the wavefunction.
3. Understanding the precise nature and use of the wavefunction is difficult. The wavefunction represents the wave nature of matter. The square of the wavefunction allows us to determine the probability distribution of the particle.
4. The Bohr model only applies to the HYDROGEN atom. Quantum mechanics provides the best description for the state of particles at this scale, but the two models have some agreement when it comes to the hydrogen atom.
5. The Heisenberg and Pauli Exclusion principle are the two biggest concepts you need to take away from this section so be sure to remember them and understand them well!

Thanks for checking out these weekly resources! Don’t forget to check out our website for group tutoring times, video tutorials and lots of other resource: www.baylor.edu/tutoring ! Answers to check you learning questions are below!

Answers: 1) d 2) $1.055 \times 10^{-24}$ 3) 18, l = 0,1,2 , m_l = 0, (-1,0,1), (-2,-1,0,1,2) m_s = (-1/2,1/2)