Week 3
MTH-1320 – PreCalculus

Hello and Welcome to the weekly resources for MTH-1320 – PreCalculus!

This week is <u>Week 3 of class</u>, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: Functions, Linear Functions, Transformations

Topic of the Week: Function Basics

Highlight 1: Compositions of Functions

Functions that have similar inputs and outputs can be added, subtracted, multiplied, and divided just as one combines "like-terms" in an equation. The written notation looks slightly different, but the meaning of it is straightforward

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(f\,/\,g)\,(x) = f(x)\ g(x)$$
$$(fg)(x) = f(x)g(x)$$

A **composition of functions** is a process through which the output of a function becomes the input of another. It is written like this:

$$(f \circ g)(x) = f(g(x))$$

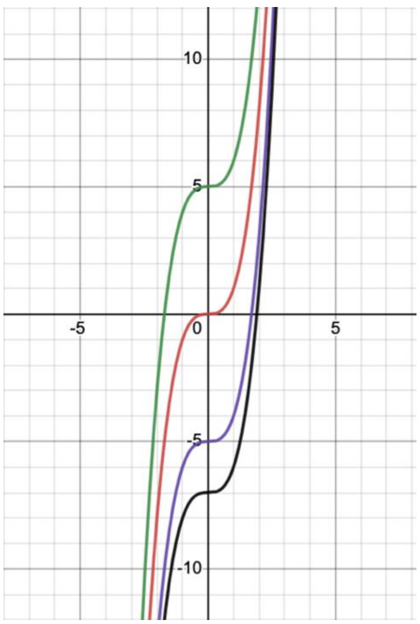
In order to evaluate a composite function, one must first evaluate the "inside" function, which in this case is g(x). After evaluating g(x), one can then plug the calculated value into f(x).

To find the domain of a composite function, we must first find the domain of f and g individually, and then exclude any points for which g(x) is outside the domain of f(x)

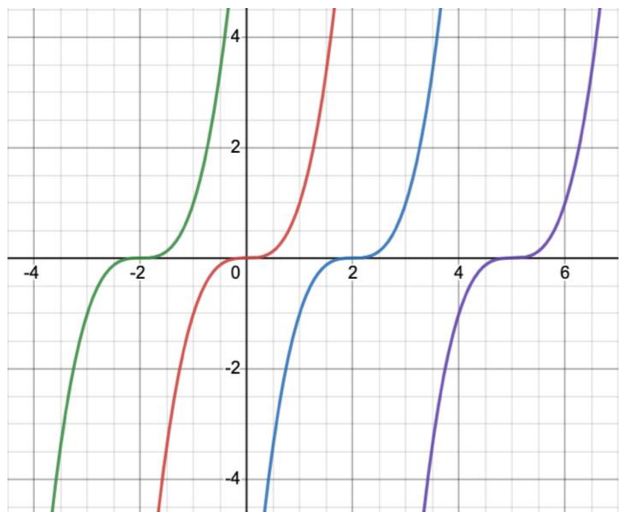
Highlight 2: Transformations of Functions

It is possible to transform and shift existing functions. It is possible to shift them up and down using vertical shifts, and to move them left and right using horizontal shifts. Vertical shifts are applied to the output, while horizontal shifts are applied to the input (they will be inside parenthesis next to the independent variable).

Given f(x) = x^3, observe what happens when we apply vertical shifts and horizontal shifts



f(x) = x^3
f(x) = x^3 + 5
f(x) = x^3 - 5
f(x) = x^3 - 7



f(x) = x^3
f(x) = $(x+2)^3$
f(x) = $(x-2)^3$
f(x) = $(x-5)^3$

*notice how the horizontal shift transformation is inside the parenthesis

Other significant transformations are :

- -f(x) = - (x^3) which shifts f(x) around the y-axis.
- f(-x) = $(-x)^3$ which shifts f(x) around the x-axis
- f(c * x) = $(cx)^3$
  - if 0 < c < 1 then f(c*x) shrinks f(x) horizontally
  - if c > 1 then f(c*x) is stretched horizontally
- c * f(x) = c * x^3

- if 0 < c < 1 then f(c*x) shrinks f(x) vertically
- if c > 1 then f(c*x) is stretched vertically

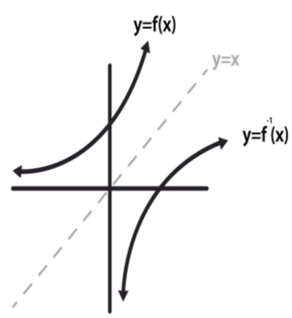
Highlight 3: Inverse Functions

The inverse function of a function f(x) takes the outputs of f(x) as inputs and returns the original input of f(x).
An inverse is marked as $f^{-1}(x)$. Be careful, the -1 is not an exponent

A function and its inverse cancel each other out.

$$f^{-1}\big(f(x)\big) = x = f(f^{-1}(x))$$

An inverse is a reflection along the line y = x.



Calcworkshop.com

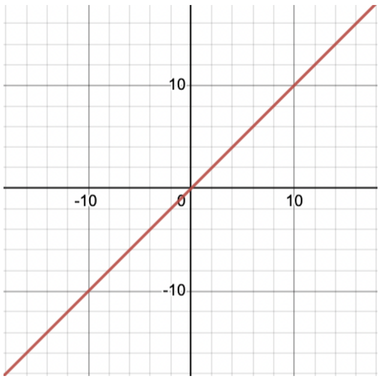
In order for a function to have an inverse, it must pass the Horizontal Line Test (meaning that if you draw an horizontal line, the graph of the function only intersects it once)

Steps to finding the inverse:
1) Solve the function in terms of x
   a. So that it is in the form y = f(x)
2) Swap x and y
   a. So that it is in the form x = f(y)
3) Solve for x again

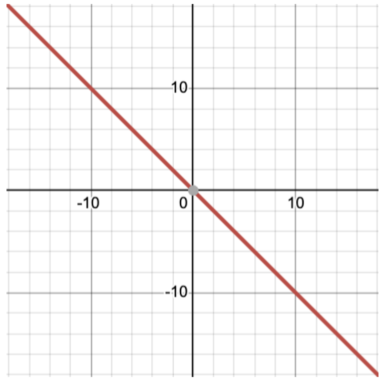
Linear Functions
A linear function is a polynomial of degree 1 (meaning the highest exponent in the function is 1)
Linear functions are usually written in **slope-intercept form**, which looks like

$$y = mx + b$$

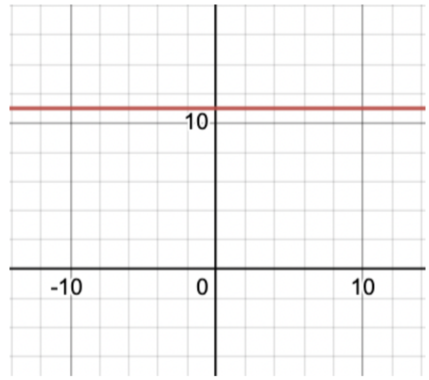
where m is the slope of the line, and b is the y intercept.
The slope of a line indicates whether the function is increasing (outputs increase as inputs increase), decreasing (outputs decrease as inputs increase), or constant (outputs are constant as inputs increase)



Increasing function.



Decreasing function



Constant function

To find the slope of a line, one must evaluate two points on a line.
Using the coordinates $(x_{1,}, y_1)$ and $(x_{2,}, y_2)$ on a certain line, the slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

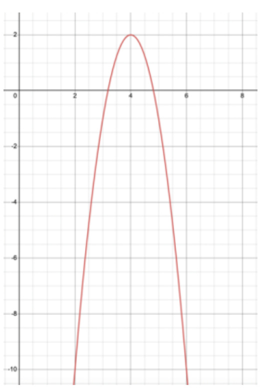
It is sometimes useful to write a line in **point-slope form**

$$(y - y_1) = m\,(x - x_1)$$

Check Your Learning

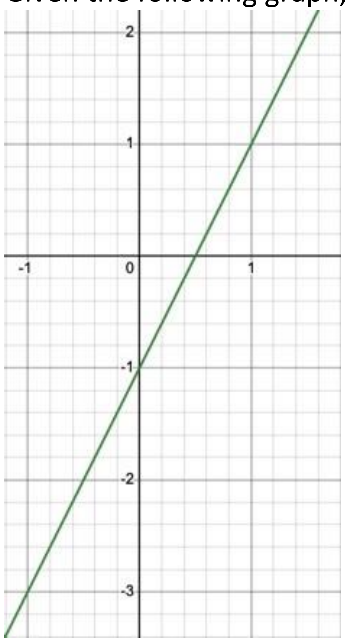
1. Let f(x) = $x^2 - 9$ and g(x) = $\sqrt{x}$. Evaluate $(f \circ g)(2)$.

2. Write the equation of the function graphed below. Be careful of any shifts or stretches!



3. Find the inverse of the function $\frac{1}{5}x + y = 7$

4. Given the following graph, find the equation of the line in slope-intercept form



Things you might struggle with:

- With composite functions, it can be confusing to figure out what to plug in first
    - Remember we always start with the inside function!
- Picturing the inverse can be tricky
    - Imagine folding a sheet of paper diagonally, what happens to the graph of the function?

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1.

Method 1:

g(x) = $\sqrt{x}$

g(2) = $\sqrt{2}$

f(x) = $x^2 - 9$

f(g(2)) = f($\sqrt{2}$) = (($\sqrt{2})^2$) – 9

= 2 – 9 = 7

Method 2:

$f \circ g$ = f(g(x))= $(\sqrt{x})^2 - 9$ = x – 9

($f \circ g$)(2) = 2 – 9 = 7

2.

By inspection alone we can tell that it is an upside-down parabola, so somewhere in our equation there will be f(x) = $-\mathrm{x}^2$.

By inspection we can see that it was shifted up by 2 -> f(x) = $-\mathrm{x}^2 + 2$

Since the parabola is not centered around 0, we can tell that there is a horizontal shift

The horizontal shift is to the right, and the magnitude of the shift is 4 -> f(x) = $-(\mathrm{x}-4)^2 + 2$

To see whether the parabola was stretched or shrunk, let's use a point on the graph (2, -10)

-10 = - a$(2-4)^2$ + 2

-10 = - a(4) + 2

-12 = -4a

a = 3

So the equation of the graph is f(x) = -3$(\mathrm{x}-4)^2 + 2$

3.

Let's follow the steps to finding the inverse:

1) Solve the function in terms of x

$$\frac{1}{5}x + y = 7$$

$$y = -\frac{1}{5}x + 7$$

2) Swap x and y

x = $-\frac{1}{5}y + 7$

3) Solve for x again

x – 7 = $-\frac{1}{5}y$

5(x – 7) = - y

-5(x – 7) = y

4.

Let's pick two points on the line, (-1, -3) and (1, 1)
Let's start by finding the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-1}{-1-1} = \frac{-4}{-2} = 2$$

Now let's find *b*, the y-intercept, using a point on the line.
So far we know the line looks like this: y = 2x + b, so if we plug in one of the points we already identified, we can solve for b

1 = 2(1) + b
1 = 2 + b
1 – 2 = b
-1 = b

So the equation of the line looks like y = 2x - 1