FINAL EXAM PREP
MTH-1320 – PreCalculus

Final exam weekly resource for MTH-1320 – PreCalculus!

This week is <u>the last week of class</u>, and typically in this week (and the surrounding weeks of class) you are reviewing for the final exam. Please use the review below (as well as other resources from the semester) as a general overview of some of the key concepts taught in the course! Please take a look at <u>all 16 weekly resources</u> listed on our website to help you review for the final exam!

If you have any questions about these study guides, the final schedule of group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website <u>www.baylor.edu/tutoring</u> or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135. **The last day of tutoring in the drop in center will be the last day of class.** To learn about additional resources available during Finals Week, please visit CASE in the West Wing basement of Sid Rich! Good luck on your final exam!

Final Review

Highlight #1: Functions

A function is a mathematical assignment of an x-value to only one y-value. You can have different x's with the same y, but never a y with different x's. You can test this through the vertical line test, meaning that there is only one point at each vertical section. A common type of function is a linear function.

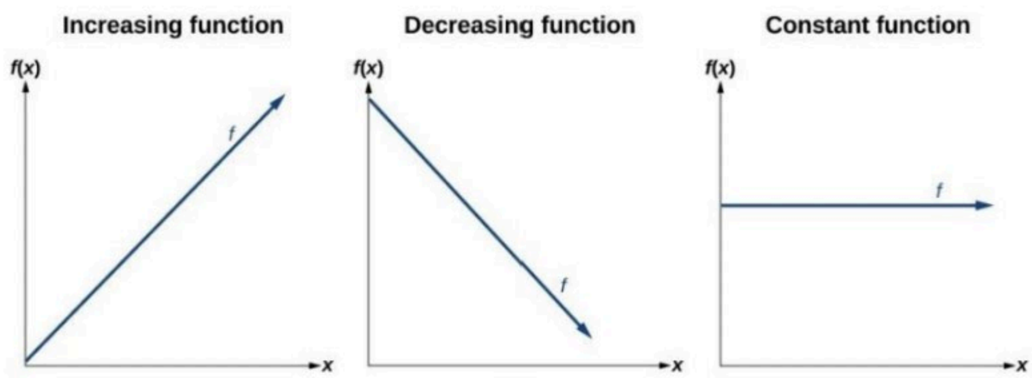
The graph of a linear function is always a straight line and therefore can be written in slope-intercept form: $f(x) = mx + b$ where m is its slope or rate of change and bis its y-intercept, the value at which it crosses the y-axis.

The slope of a linear function determines whether it is increasing, decreasing, or constant.

For **increasing** functions, the outputs increase with the inputs, and the graph has a positive slope.

For **decreasing** functions, the outputs decrease as the inputs increase, and the graph has a negative slope.

A **constant** function ($f(x) = c$) outputs only one value, no matter the input. Its graph is therefore a horizontal line with a slope of zero.



Quadratic Functions

A quadratic function is a function of degree 2, and its graph resembles a parabola. The **general form** of a quadratic equation is

$$f(x) = ax^2 + bx + c$$

Where a, b, c are real constants and a ≠ 0 (otherwise the function wouldn't be of degree 2).

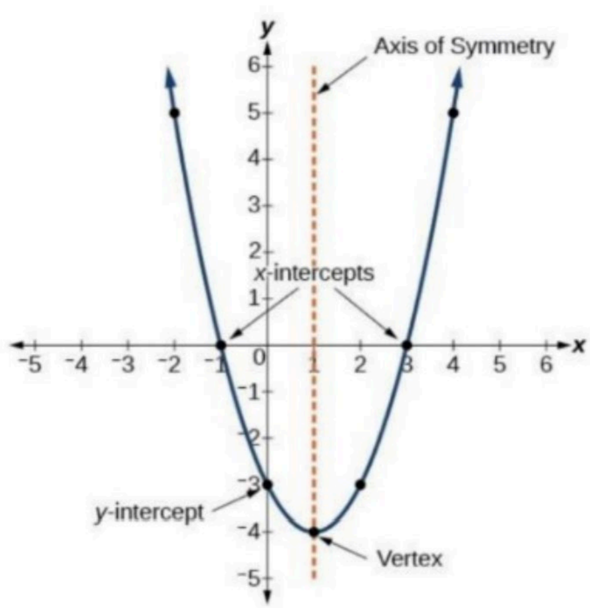
Parabolas are easier to recognize (and graph) when written in their **standard form**

$$f(x) = a(x-h)^2 + k$$

Where (h, k) is the vertex of the parabola (either it's lowest or highest point).
If a > 0 the parabola opens upward, if a < 0 it opens downward

To find the x-intercepts of a parabola, you can use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Here is the graph of a parabola



Polynomial Functions

A polynomial function is a function that is smooth, doesn't have points or corners (known as cusps), and has no breaks (which means the graph is continuous).
Note! The domain of a polynomial is all real numbers.

In order to find the zeroes (x-intercepts) of a polynomial, it is first necessary to factor it. There are different techniques to factoring polynomials:

- Factoring out the greatest common factor (the biggest power of x that is present in all terms)
- Factoring binomials that are difference of squares ($x^2 - 16 = (x+4)(x-4)$)
- If a polynomial has 4 terms, it can be useful to group terms in groups of two, factor our the greatest common factor in each group, and then pull out the greatest common factor between the two groups

In order to divide polynomials, one might have to use long division. The steps are outlined below

Long division

The following steps show how to perform long division

1) Place the numerator (dividend) under the long division symbol $\overline{)\ }$. If the polynomial "skips" any powers write them in with a coefficient of 0.
2) Place the denominator (divisor) to the left of the long division symbol $\overline{)\ }$
3) Divide the first term of the dividend by the first term of the divisor, writing the quotient on top of the long division symbol $\overline{)\ }$
4) Multiply the quotient term by the whole divisor and write it below the dividend
5) Subtract the product from the original dividend (the first term should cancel out)
6) Pull down the next term of the original dividend
7) Repeat steps 3-7 for all following terms of the dividend
8) Divide the remainder by the divisor and add it to the rest of the quotient

Finding vertical asymptotes and removable discontinuities

The mathematical definition of a vertical asymptote is "a vertical line where the graph tends towards infinity as the input approaches a".
Note! Vertical asymptotes are caused when the denominator of a rational function equals 0 (as we are dividing by 0, which leads to the output being undefined).
However, be careful. Sometimes, certain outputs that lead to division by 0, do not cause a vertical asymptote, but cause a **removable discontinuity.**
How do we tell the difference between an input that causes a vertical asymptote and one that causes a removable discontinuity?

1) Factor the numerator and denominator
2) If a factor appears in both the numerator and denominator, there exists a removable discontinuity at that value of x

3) If a factor only appears in the denominator, there exists a vertical asymptote at that value of x

Finding horizontal and slant asymptotes

The mathematical definition of a horizontal asymptote is a “horizontal line y = b where the graph approaches the line as inputs increase or decrease without bound”. Horizontal and slant asymptotes are equivalent to a function’s end behavior and are therefore determined by the leading term.

There are three cases that occur with horizontal and slant asymptotes

1) The degree of the numerator is less than the degree of the denominator: horizontal asymptote at y = 0
2) The degree of the numerator is greater than the degree of the denominator: no horizontal asymptote, slant asymptote
3) The degree of the numerator is equal to the degree of the denominator: horizontal asymptote at y = ratio of leading coefficients

A slant asymptote is defined by a linear equation: in order to find the linear equation use long division or synthetic division. The quotient (without the remainder) is the equation of the line of the slant asymptote.

Remember to find the domain and range of functions! Take note of any numbers that might cause division by 0 or square roots of negative numbers.

Highlight #2: Exponential and Logs

Exponential Functions model outputs that change at a rate proportional to the current quantity, and the general form is

$$f(x) = ab^x$$

Where a is the initial value (x = 0) and b is the **base** or the **growth factor.**

Both a and b have restrictions:

- a cannot equal 0 (otherwise the term cancels out and f(x) = 0)
- b must be positive (else the output oscillates)
- b cannot equal 1 (otherwise f(x) = a, since 1 to any power is still 1)
- a and b must be real numbers

Things to note about exponential functions:

- Domain: (-∞, ∞)
- Range:
  - (0, ∞) if a is positive
  - (-∞, 0) if a is negative
- Y-intercept at point (0, a)

There is an horizontal asymptote at y =0

The constant e

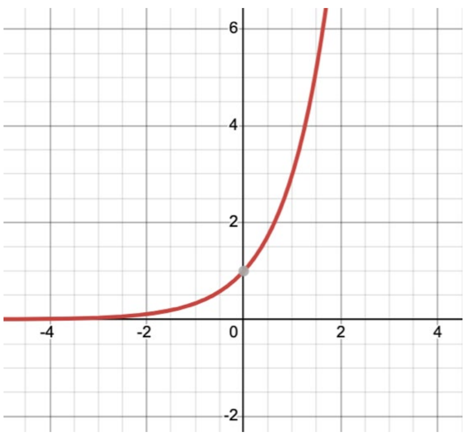
There is a base that is very often used in mathematics. It is denoted as e, and it represents the number 2.72.

The mathematical definition of e is: as n -> ∞, $(1 + \frac{1}{n})^n$ -> e

The function e^x is called the natural exponential.

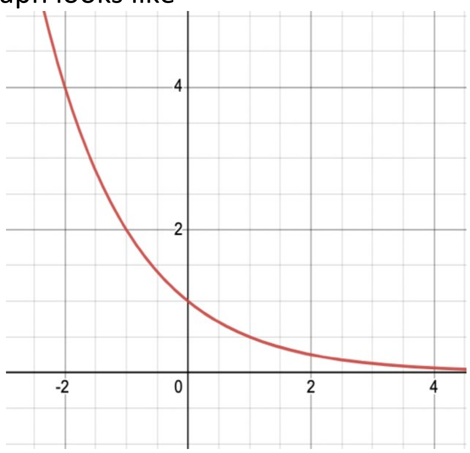
Note! Although it may look like a variable, e is just a constant!

This is what an exponential function graph looks like



f(x) = b^x

b > 1



f(x) = b^x

0 < b < 1

Logarithms

The inverse of an exponential function is called a logarithmic function. So if f(x) is an exponential function of the form $f(x) = b^x$, its inverse has the form $f^{-1}(x) = \log_b(x)$.
A logarithmic function reverses an exponential function.

In fact,

$y = \log_b(x)$ and $x = b^y$ are equivalent under certain conditions

- x must be greater than 0
- b must be greater than 0 (this was also a requirement for exponential functions)
- b cannot be equal to 1 (also a requirement with exponential functions)

Since logs are the inverse of exponential functions, the domain of a log function is the range of its corresponding exponential, and its range it's the domain of the exponential. So

- Domain: (0, ∞)
- Range: (-∞, ∞)

There are two common logarithms that have their own special notation:

- Logarithms with base 10 are often written as $\log x$ (notice how there is no tiny b at the side of the log). Therefore, if you see a log with no space for b, you must assume that the log is base 10
- Logarithms with base e are written as ln(x).

Name of Property	Exponential Version	Logarithmic Version
	$b^0 = 1$	$\log_b(1) = 0$
	$b^1 = b$	$\log_b(b) = 1$
Inverse property	$b^{\log_b x} = x \quad (x > 0)$	$\log_b b^x = x$
One-to-one property	$b^x = b^y$ if and only if $x = y$	$\log_b(M) = \log_b(N)$ if and only if $M = N$
Product rule	$b^x b^y = b^{x+y}$	$\log_b(MN) = \log_b(M) + \log_b(N)$
Quotient rule	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$
Power rule	$(b^x)^y = b^{xy}$	$\log_b(M^n) = n\log_b(M)$

Change of base formula

Often we need to transform existing logarithms into logarithms with a different base. We can do so by using the change of base formula, which states

$$\log_b(M) = \log_n(M) \log_n(b)$$

Where n is the new base we want to use.

Highlight #3: Angles

An angle is "the union of two rays having a common endpoint". Each ray starts at the endpoint and extends in a straight line from it out to infinity. The endpoint is also known as the angle's **vertex**.

When we draw angles, it is the convention to draw them in standard position, in which the vertex lies at the origin of the coordinate plane and the initial side lies on the positive x-axis.

Angles can be positive or negative. Positive angles are measured in the counterclockwise direction, and negative angles are measured in the clockwise direction.

We can measure angles in 2 ways: angles and radians. There exists a relationship between degrees and radians, so we can easily change measurements from one unit to the other if it helps with solving a problem

The relationship is as follows: 180° = π radians

Note! If no unit is given, it is implied that the units are radians

Coterminal angles are “two angles in standard position that have the same terminal side”. For a particular angle that is not between 0° (0 radians) and 360° (2� radians), we often want to find the coterminal angle between 0° (0 radians) and 360° (2� radians) because this range is easy to work with.

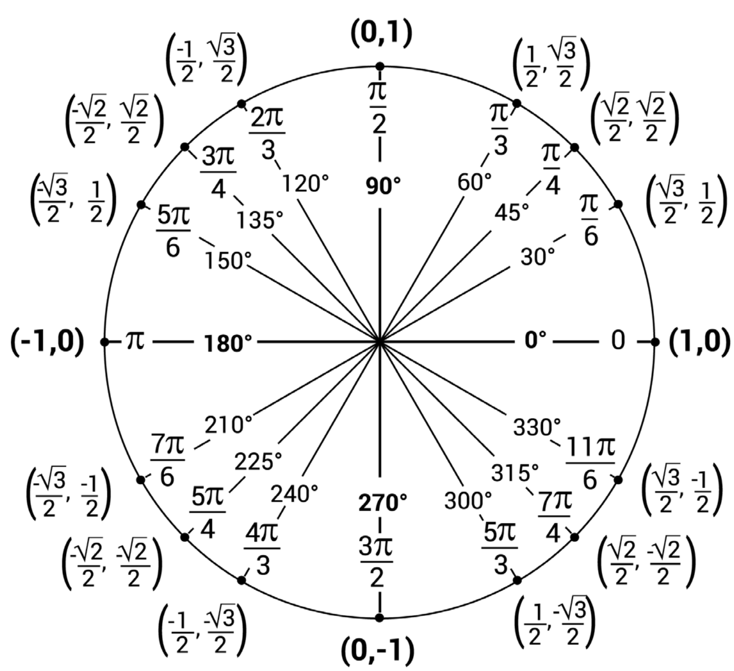
Application of Angles

Angles can be applied to many applications involving circles and rotational motion.

1. **Arc Length**
   An arc is a portion of the outline of a circle. The formula for arc length s is $s = r\theta$ where r is the radius of the circle that the arc is part of, and θ is the measure in radians of the angle that forms the arc.
2. **Area of a Sector**
   A sector is “a region of a circle bounded by two radii and the intercepted arc, like a slice of pizza or pie. To find a sector’s area, we can multiply the whole circle’s area (πr^2) by the fraction of the circle that the sector is.
   This results in the formula: $A = \frac{1}{2} * r^2 * \theta$ (where θ is the angle defining the sector)
   Note that θ must be in radians for the equation to be valid.

Highlight #4: Trig functions

The most relevant part of trig functions is the unit circle, as it gives us a lot of useful information.



If we call one of these angles θ, the point (x, y) at which the terminal ray of the angle intersects the unit circle is given by

$$x = \cos\theta \text{ and } y = \sin\theta$$

$f(\theta) = \cos\theta$ is **the cosine function**, and $f(\theta) = \sin\theta$ is the **sine function**.
Note that these functions' domain is all real numbers, and their range is $-1 \leq \theta \leq 1$.

An important identity relating sine and cosine is the Pythagorean Identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

There are four other trigonometric functions: tangent, secant, cosecant, and cotangent. Either a point on the unit circle or the values of sine and cosine for a particular angle can be used to find the values of the other four trigonometric functions

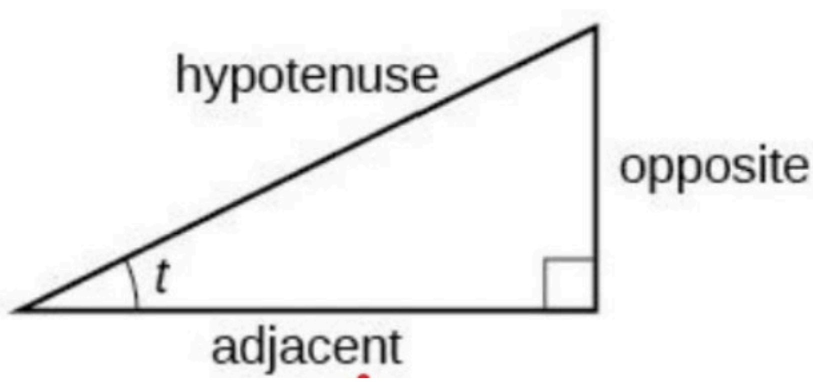
Name	Notation	Definition in Terms of Coordinates on the Unit Circle	Relationship to Other Trigonometric Functions
Cosine	$\cos\theta$	x	
Sine	$\sin\theta$	y	
Tangent	$\tan\theta$	$\frac{y}{x}, x \neq 0$	$\frac{\sin\theta}{\cos\theta}$
Secant	$\sec\theta$	$\frac{1}{x}, x \neq 0$	$\frac{1}{\cos\theta}$
Cosecant	$\csc\theta$	$\frac{1}{y}, y \neq 0$	$\frac{1}{\sin\theta}$
Cotangent	$\cot\theta$	$\frac{x}{y}, y \neq 0$	$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Functions can be even, odd, or neither. Remember that even functions are symmetric around the y-axis, and odd functions are symmetric around the origin.
If a function is even, then $f(x) = f(-x)$, and if a function is odd, then $f(-x) = -f(x)$.

How is this relevant to trigonometric functions?
- $\cos(-\theta) = \cos\theta$ (even)
- $\sin(-\theta) = -\sin\theta$ (odd)
- $\tan(-\theta) = -\tan\theta$ (odd)
- $\sec(-\theta) = \sec\theta$ (even)
- $\csc(-\theta) = -\csc\theta$ (odd)

• cot(−θ) = − cot θ (odd)

Some important applications of trigonometry involve **right triangles**. So far, we have defined the trigonometric functions in terms of a point on the unit circle, but we can also define them based on the sides of a right triangle. This makes the trigonometric functions much more versatile. The sides of a right triangle are called the hypotenuse, adjacent side, and opposite side. The hypotenuse is always the angle opposite to the right angle. The opposite and adjacent sides, however, vary based on which acute angle.



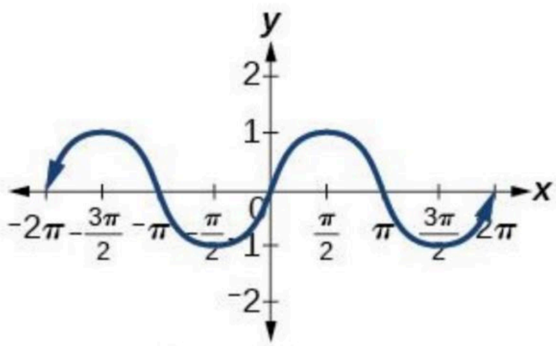
Knowing these terms for the sides of a right triangle, we can now learn how trigonometric functions of an acute angle θ are related to these sides.

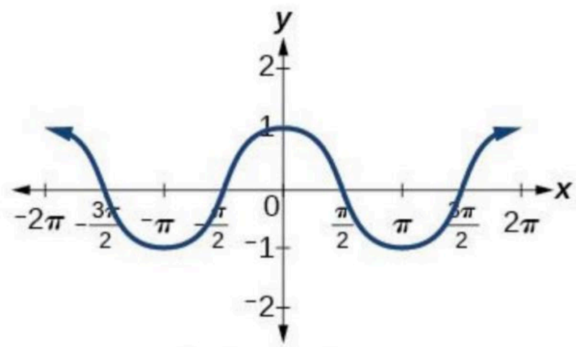
• sin(θ) = opposite/hypothenuse
• cos(θ) = adjacent/hypotenuse
• tan(θ) = opposite/adjacent

A way to remember these relationships is the mnemonic Soh Cah Toa:
• Soh: Sine is opposite over hypotenuse.
• Cah: Cosine is adjacent over hypotenuse.
• Toa: Tangent is opposite over adjacent. in the triangle we want to examine.

The opposite side is the side opposite of the acute angle in question. The adjacent side is the side adjacent to (next to) the acute angle in question.

The sine and cosine functions oscillate like waves.





A function that can be described as a combination of transformations of the sine or cosine function is called a **sinusoidal function** or simply a sinusoid.

Sinusoids have the general form

y = A sin(Bx-C) + D or y = A cos(Bx -C) + D

Trigonometric functions and their inverses

To determine the value of an inverse trigonometric function, you can use an approach similar to the approach used for logarithmic functions, which are inverses of exponential functions. For example, if you are given an inverse sine function, y = sin−1x, ask yourself, "What angle � results in sin x = y?"

Inverse Trigonometric Function Name	**Notation**	**Domain**	**Range**
Inverse Sine or Arcsine Function	$y = \sin^{-1} x$ or $\arcsin x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
Inverse Cosine or Arccosine Function	$y = \cos^{-1} x$ or $\arccos x$	$[-1, 1]$	$[0, \pi]$
Inverse Tangent or Arctangent Function	$y = \tan^{-1} x$ or $\arctan x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

We can use the double-angle formulas to rewrite a trigonometric function of a double angle 2θ as a combination of trigonometric functions of the angle θ. We can also use the reduction formulas "to reduce the power of a given expression involving even powers of sine or cosine". Lastly, we can use the half-angle formulas to rewrite a trigonometric function of a half angle $\theta/2$ with trigonometric functions of θ.

Trigonometric Function	Double-Angle Formula	Half-Angle Formula	Reduction Formula
Cosine	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 1 - 2\sin^2\theta$ $= 2\cos^2\theta - 1$	$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$
Sine	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$	$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$
Tangent	$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$	$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $= \frac{\sin\theta}{1+\cos\theta}$	$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Tips for the Final

1. There is a ton of stuff you learned all semester! Try to split it up into sections and mix it up! The problems on the final won't follow an order, so don't get your brain used to it!
2. Make sure to go over old homework. Since there will be a limited amount of time to study, don't do problems that you already know how to do.
3. At a certain point, it will be more beneficial to sleep than it will be to study, make sure you take care of your health for finals week!

Thanks for using these resources this semester! Best wishes on your final exam!