Week 11
MTH-1320 – PreCalculus

Hello and Welcome to the weekly resources for MTH-1320 – PreCalculus!

This week is Week 11 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: angles, unit circle, sine and cosine functions

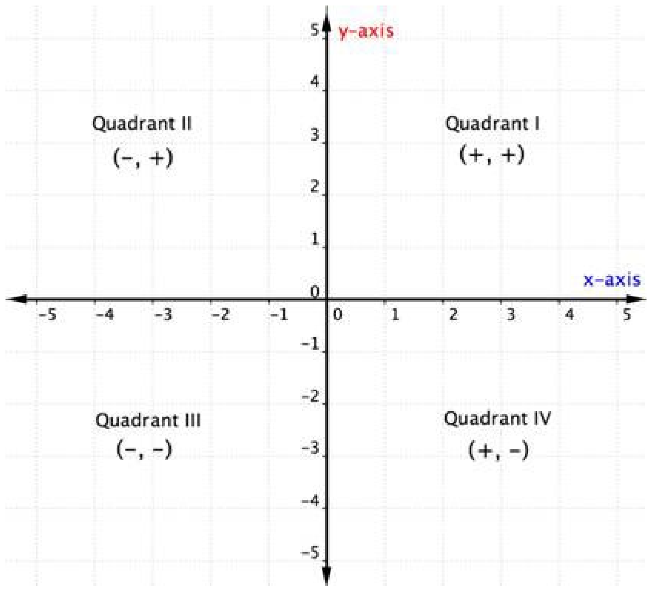
Topic of the Week: All about Angles!

An angle is "the union of two rays having a common endpoint". Each ray starts at the endpoint and extends in a straight line from it out to infinity. The endpoint is also known as the angle's **vertex**.

When we draw angles, it is the convention to draw them in standard position, in which the vertex lies at the origin of the coordinate plane and the initial side lies on the positive x-axis.

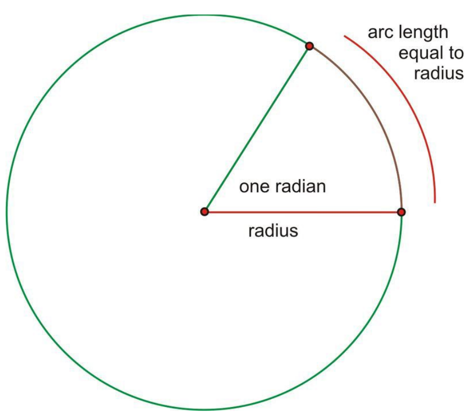
Angles can be positive or negative. Positive angles are measured in the counterclockwise direction, and negative angles are measured in the clockwise direction.

When discussing trigonometry, we must know how to describe the four quadrants of the coordinate plane. The quadrants are numbered in the counterclockwise direction as shown below.



Highlight #1: Drawing and Measuring Angles

The measure of an angle is "the amount of rotation from the initial side to the terminal side". Angles can be measured using 2 different units : degrees and radians.
A degree is "1/360 of a circular rotation" and an angle measure in degrees is marked with the symbol °.
A radian is "the measure of a central angle of a circle that intercepts an arc equal in length to the radius of that circle". This means that when a 1-radian angle is drawn in standard position, and the tips of the rays touch a circle centered around the origin, the arc connecting the initial and terminal sides of the angle is equal in length to the radius of the circle.



Here is a drawn representation of the relation of a radian to a circle of an arbitrary radius

There exists a relationship between degrees and radians, so we can easily change measurements from one unit to the other if it helps with solving a problem

The relationship is as follows: 180° = π radians

Note! If no unit is given, it is implied that the units are radians

Example 1
Convert degrees and radians

a. 60°

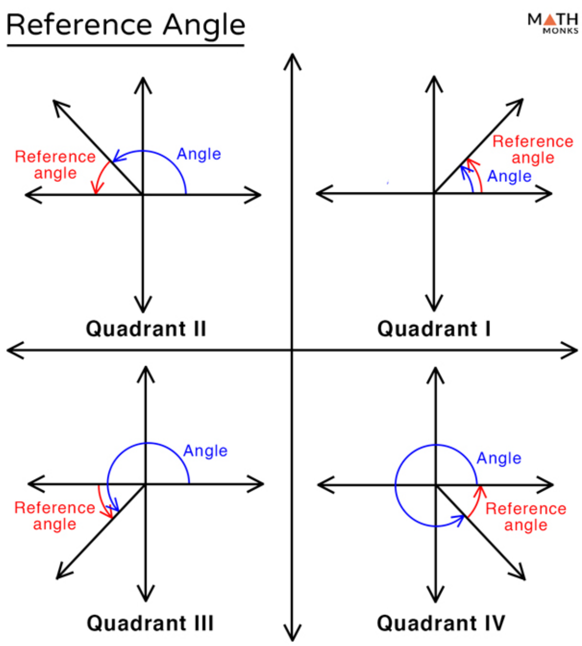
b. 5π

Drawing angles in standard position

1. Divide the given angle measure by 360° or 2π radians, depending on the given units.
2. Reduce the fraction.
3. Rewrite the reduced fraction so that you can visualize the portion of the circle that it represents.
4. Draw the initial side of the angle on the positive x-axis.
5. Rotate counterclockwise if the angle is positive and clockwise if the angle is negative.
6. Draw the terminal side so that the angle contains the fraction that you calculated.

Coterminal angles
We call two angles in standard position that have the same terminal side **coterminal angles**. Coterminal angles occur because angles rotate in a circle. Therefore, if we are given an angle greater than 360° (2π radians), we can find the coterminal angle between 0° (0 radians) and 360° (2π radians) by subtracting 360° (2π radians) from the given angle until the angle is less than 360° (2π radians).
If we are given an angle less than 360° (2π radians), we can find the coterminal angle between 0° (0 radians) and 360° (2π radians) by adding 360° (2π radians) to the given angle until the angle is greater than 0° (0 radians).

Reference angles
An angle's reference angle is the angle in standard position that has the same dimension as the smallest acute angle formed by the terminal side of an angle and the x-axis.

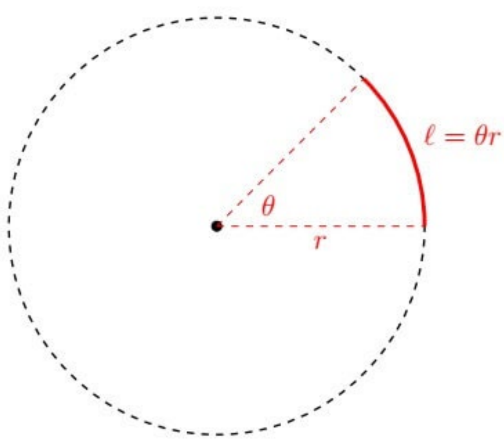


The reference angles in the different quadrants are calculated as follows:
<u>Quadrant 1:</u> the angle IS the reference angle
<u>Quadrant 2</u>: π – angle, 180° - angle
<u>Quadrant 3:</u> angle - π, angle - 180°
<u>Quadrant 4</u>: 2π – angle, 360° - angle

Highlight #2: Application of Angles

Angles can be applied to many applications involving circles and rotational motion.

1. **Arc Length**
   An arc is a portion of the outline of a circle. The formula for arc length s is $s = r\theta$ where r is the radius of the circle that the arc is part of, and θ is the measure in radians of the angle that forms the arc.

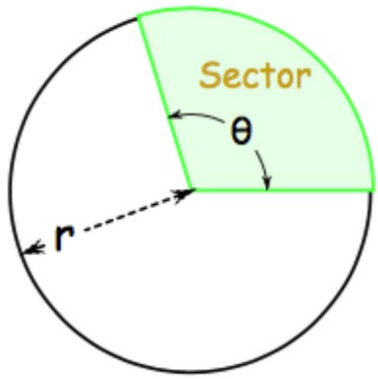


2. Area of a Sector

A sector is "a region of a circle bounded by two radii and the intercepted arc, like a slice of pizza or pie. To find a sector's area, we can multiply the whole circle's area (πr^2) by the fraction of the circle that the sector is.

This results in the formula: $A = \frac{1}{2} * r^2 * \theta$ (where θ is the angle defining the sector)

Note that θ must be in radians for the equation to be valid.



3. Linear and Angular Speed

When an object moves in a circle, it is said to be in rotational motion. An object in rotational motion has linear speed v or "speed along a straight path" like all objects in motion. However, it also has angular speed ω, which only objects in rotation motion have. The table gives the formulas for linear speed, angular speed, and the relationship between the two.

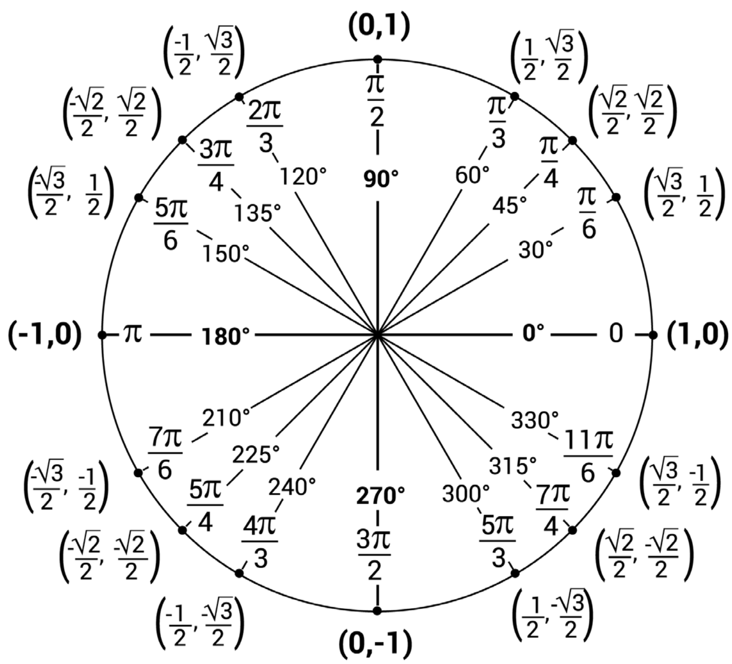
Linear Speed	$v = \frac{s}{t}$
Angular Speed	$\omega = \frac{\theta}{t}$
Relationship Between Linear and Angular Speed	$v = r\omega$

Example 2

A circle has a radius of 5 and an angle of 60°. What is the arc length? What is the area of the sector?

Highlight #3: the Unit Circle: Sine and Cosine Functions

The unit circle is the circle with radius 1 centered at the origin.
The unit circle with frequently used angles and the x- and y-coordinates where the angles' terminal sides intersect the circle. Additionally, it is annotated with the signs of the x and y values in each quadrant.
<u>I highly recommend memorizing it, at least the first quadrant, from which all the other quadrants can be found.</u>



If we call one of these angles θ, the point (x, y) at which the terminal ray of the angle intersects the unit circle is given by

$$x = \cos\theta \text{ and } y = \sin\theta$$

$f(\theta) = \cos\theta$ is **the cosine function**, and $f(\theta) = \sin\theta$ is the **sine function**.
Note that these functions' domain is all real numbers, and their range is $-1 \le \theta \le 1$.

An important identity relating sine and cosine is the Pythagorean Identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

This identity comes from the equation for the unit circle, $x^2 + y^2 = 1$.

Example 3

What is the sin and cos of the angle (don't use a calculator!)

a. 30°
b. π

Things you might struggle with:

- Memorizing the unit circle (it is painstaking, but useful)
- Finding reference angle
  - First identify the quadrant, then use the appropriate formula

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Check your Learning

1. Convert degrees and radians.
   a. 180°

   b. -60°

   c. π

   d. -3π

2. What is the arc length? This circle has a radius of 2 and an angle of -90°

3. What is the area of the sector? The circle has radius of 10 and an angle of 30°

Solutions to the Examples

1.

a. $60° * \frac{\pi}{180°} = \frac{\pi}{3}$

b. $5\pi * \frac{180°}{\pi}$ = 900°

2. A circle has a radius of 5 and an angle of 60°. What is the arc length? What is the area of the sector?

Let's start with the arc length:

$$s = r * \theta \text{ where } \theta \text{ is in radians}$$

Let's convert 60° to radians

$$60° * \frac{\pi}{180°} = \frac{\pi}{3}$$
$$s = 5 * \frac{\pi}{3} = \frac{5\pi}{3}$$

Let's calculate the area of the sector:

$$A = \frac{1}{2} * r^2 * \theta$$

$$A = \frac{1}{2} * (5)^2 * \frac{\pi}{3} = \frac{25\pi}{6}$$

3.

a. $\sin(30°) = \frac{\sqrt{3}}{2}$ $\cos(30°) = \frac{1}{2}$
b. $\sin(\pi) = 1$ $\cos(\pi) = 0$

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1.

a. 180 degrees to radians 180 * π / 180 = π
b. -60 degrees to radians -60 * π / 180 = -π / 3
c. π radians to degrees π * 180 / π = 180°
d. -3π to degrees -3π * 180 / π = -520° or 180° (coterminal!)

2.

First you have to change the angle to radians

-90 degrees = π / 2

Now just plug into the equation!

$$s = r\theta$$

$$s = 2 * \pi / 2$$

s = π (note that this value is not negative, length is always positive!)

3.

30 degrees = π / 3

Now just plug into the equation!

$$A = ½ * \theta * r2$$

$$A = ½ * \pi / 3 * (102)$$

$$A = \pi / 6 * 100$$

$$A = 50\pi / 3$$

Pictures taken from:

https://content.nroc.org/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U13_L1_T1_text_final.html

https://www.ck12.org/c/trigonometry/radian-measure/lesson/Radian-Measure-TRIG/