Week 4
MTH-1320 – PreCalculus

Hello and Welcome to the weekly resources for MTH-1320–PreCalculus!

This week is Week 4 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: Linear Functions, Graphs of Linear Functions, Modeling, Quadratic functions

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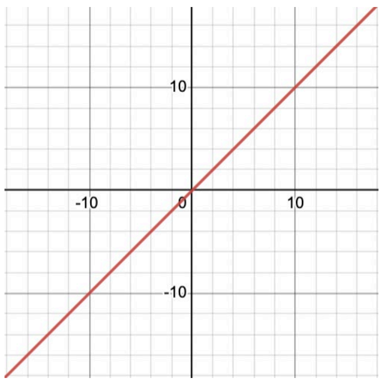
Topic of the Week: Linear Functions

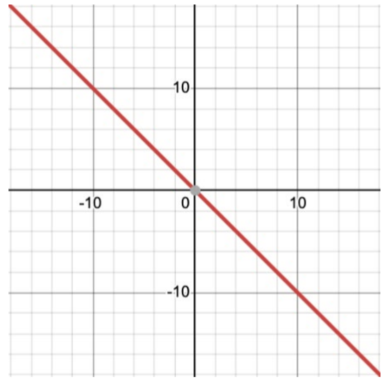
A linear function is a function with a constant rate of change, which is equivalent to saying a polynomial of degree 1 (meaning the highest exponent in the function is 1)
Linear functions are usually written in **slope-intercept form**, which looks like

$$y = mx + b$$

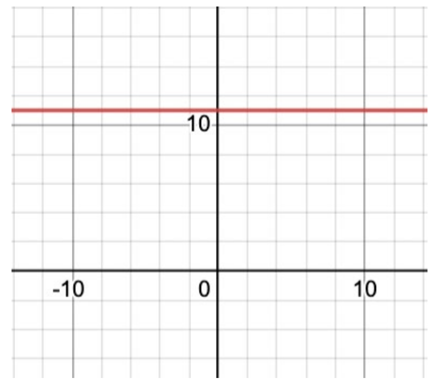
where m is the slope of the line, and b is the y intercept.
The slope of a line indicates whether the function is increasing (outputs increase as inputs increase), decreasing (outputs decrease as inputs increase), or constant (outputs are constant as inputs increase)



Increasing function.



Decreasing function



Constant function

To find the slope of a line, one must evaluate two points on a line.
Using the coordinates $(x_{1,}, y_1)$ and $(x_{2,}, y_2)$ on a certain line, the slope of the line is

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

It is sometimes useful to write a line in **point-slope form**

$$(y - y_1) = m\,(x - x_1)$$

Highlight 1: Graphing Linear Functions

There are 3 different methods to graphing linear functions

Method 1: Plotting points

1. Choose two or more input values (ideally, something that is easy to graph like integers or defined fractions)
2. Evaluate the function at each input value
3. Plot each point and its output on a coordinate plane
4. Draw a straight line to connect the points

Method 2: Using the slope and y-intercept

If a linear function is given to you in the form y = mx + b, this method is the most convenient

1) Identify the y intercept (b) of the function
2) Plot the point (0,b) on the coordinate plane
3) Identify the slope (m) of the function
4) Determine the change of y and the change in x, using the formula

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

5) Starting from the fixed point (0,b) rise by $y_2 - y_1$ units and move right by $x_2 - x_1$ units

6) Plot a point
7) Starting from the new point, rise by $y_2 - y_1$ units and move right by $x_2 - x_1$ units
8) Draw a straight line through all the points

If the linear function is written in point-slope form, $(y - y_1) = m(x - x_1)$, use the given point (x_1, y_1) as the starting point

Method 3: Transforming the Identity Function

An alternative method when the function is written in slope-intercept form, is to use transformations on the identity function

1) Graph the identity function, y = x
2) Identify and apply and vertical stretch/compression to y = x based on the slope of the function
3) Vertically shift the graph y = mx by b units, where b is the y-intercept of the function

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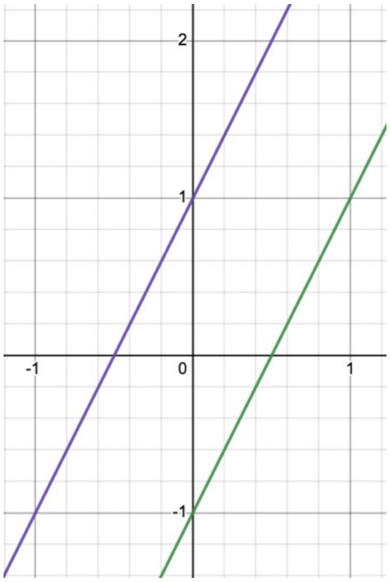
Highlight 2: significant wording

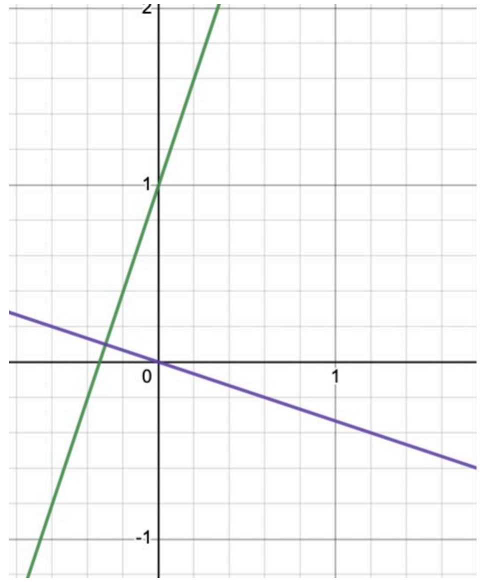
- x-intercept: the point at which a function crosses the x -axis. To find the x-intercept, set the function to 0 and solve for x
- Horizontal lines: have a constant graph (image on page 1). The equation of a horizontal line is y = b, since their slope is 0 (which causes the mx term to disappear)
- Vertical lines: have only one input and infinitely many outputs. Since they do not pass the vertical line test, vertical lines are not functions. Since there is no change in x, their slope is undefined
- Parallel lines: are lines that never intersect. In order for them to never meet, they must have the same slope, so given lines f(x) and g(x), m_1 and m_2 are the same. If both the slope (m) and the y-intercept (b) are the same, f(x) and g(x) are called coincident (they are the same line!)
- Perpendicular lines: are lines that intersect with an angle of 90 °. They are also called "orthogonal". Two lines are perpendicular if their slopes are the negative reciprocal of each other



Parallel lines

$y = 2x - 1$

$y = 2x + 1$



Perpendicular lines

$y = 3x + 1$

$y = -\frac{1}{3}x$

Highlight 3: Solutions to a system of equations

To find the solution to a system of equation means to find the x coordinate at which the two lines intersect. There are 3 easy steps to do so

1) Set the two functions equal to each other
2) Solve for x (get x alone by itself on one side of the equation)
3) Plug the evaluated x into one of the original functions to find the corresponding y value

Modeling with Linear Functions

It is important to know how to model real-life situations using linear models. Given a word problem, look for certain words:

- An initial condition is the y-intercept
- Words like “per”, “every”, or “each” indicate the slope (for example “every year the revenue increases by 10k” means that the slope is 10k)
- Sometimes the prompt will present specific points (such as “in 2010, the revenue was 10k, and in 2015 the revenue was 20k”). By using those points, you can find the slope and the y-intercept
- If time is one of the variables, it is often the input

Highlight 4: Quadratic functions

A quadratic function is a function of degree 2, and its graph resembles a parabola. The **general form** of a quadratic equation is

$$f(x) = ax^2 + bx + c$$

Where a, b, c are real constants and a ≠ 0 (otherwise the function wouldn't be of degree 2).

Parabolas are easier to recognize (and graph) when written in their **standard form**

$$f(x) = a(x-h)^2 + k$$

Where (h, k) is the vertex of the parabola (either it's lowest or highest point).
If a > 0 the parabola opens upward, if a < 0 it opens downward

Finding the vertex of a parabola:

1) Calculate the x coordinate using the formula

$$h = \frac{-b}{2a}$$

2) Evaluate f(x) at x = h to find k

Finding the roots (x-intercepts) of a parabola:

- Method 1:
  1) Factor the general form of the equation
  2) Set each factor to 0
  3) Solve for x
- Method 2:
  1) Use the quadratic formula using the values of a, b, c from the general form of the equation

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  2) Don't forget to evaluate the plus or minus!
- Method 3:
  1) Rewrite quadratic equation in standard form
  2) Set it equal to 0
  3) Solve for x (very similar to completing the square

Check Your Learning

1. Solve the following systems of equations

f(x) = 3(x+2) + 5
g(x) = 5x + 7

2. Write the following quadratic function in standard form: f(x) = $2x^2 + 2x - 3$

Things you might struggle with:

- Interpreting a word problem and using a line to model the prompt
    - Remember the key words we mentioned!
- Going from the general form of a quadratic equation to the standard form

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Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to the Check Your Learning

1.

f(x) = 3(x+2) + 5
g(x) = 5x + 7

1) Set the two functions equal to each other

   f(x) = g(x)
   3(x+2) + 5 = 5x + 7

2) Solve for x (get x alone by itself on one side of the equation)

   3x + 6 + 5 = 5x + 7
   3x + 11 = 5x + 7
   4 = 2x
   x = 2

3) Plug the evaluated x into one of the original functions to find the corresponding y value

   g(2) = 5(2) + 7
   g(2) = 17

   So the two lines intersect at the point (2, 17)

2.

To convert this quadratic in standard form we must find h and k

h = $\frac{-b}{2a}$ = $\frac{-2}{2(2)}$ = -0.5
k = f(-0.5) = $2(0.5)^2\ + 2(0.5) - 3$
= -3.5
So the function can be rewritten as f(x) = $2(x - (-\ 0.5))^2\ - 3.5$
f(x) = $2(x +\ 0.5)^2\ - 3.5$