Week 6
MTH-1320 – PreCalculus

Hello and Welcome to the weekly resources for MTH-1320 – PreCalculus!

This week is Week 6 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: Graphs of polynomials, factoring, intermediate value theorem, dividing polynomials

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Topic of the Week: Polynomial Functions

A polynomial function is a function that is smooth, doesn't have points or corners (known as cusps), and has no breaks (which means the graph is continuous).
Note! The domain of a polynomial is all real numbers.

Highlight #1: Graphing polynomials

There are 7 easy steps to graphing polynomials:

1) Find the x and y intercepts
2) Check for symmetry
    a. An even function is symmetric around the y axis, so f(x) = f(-x)
    b. An odd function is symmetric around the origin so f(-x) = -f(x)
3) Identify the multiplicity of the zeroes and graph the behavior at the zeroes
4) Determine the end-behavior by examining the leading term
5) Graph a continuous graph across all the points
6) Ensure the turning points are 1 less than the degree of the polynomial
7) Check the graph with technology (optional)

Highlight #2: Factoring Techniques

In order to find the zeroes of a polynomial, it is first necessary to factor it. There are different techniques to factoring polynomials:

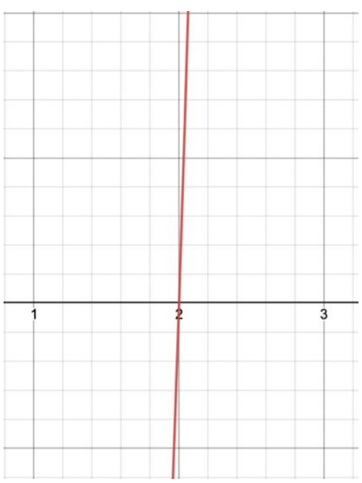
- Factoring out the greatest common factor (the biggest power of x that is present in all terms)
- Factoring binomials that are difference of squares ($x^2 - 16 = (x+4)(x-4)$)
- If a polynomial has 4 terms, it can be useful to group terms in groups of two, factor our the greatest common factor in each group, and then pull out the greatest common factor between the two groups
  - Ex. $x^3 + 2x^2 + 3x + 6$
  - $(x^3 + 2x^2) + (3x + 6)$ (Dividing in 2 groups)
  - $x^2(x + 2) + 3(x+2)$ (Factoring greatest common factor of each group)
  - $(x+2)(x^2 + 3)$ (Pulling out greatest common factor between two groups)

Highlight #3: Multiplicities

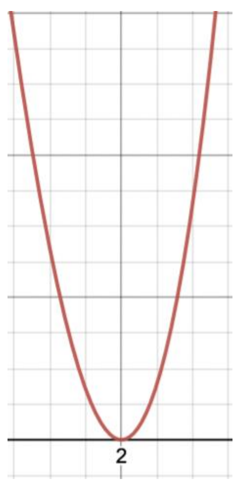
The multiplicity of a factor is the number of times a certain factor appears in a factored equation. The multiplicity is denoted by the exponent over a certain factor.

- If a zero has a multiplicity of 1, then the graph crosses the x-axis at that zero in a straight line
- If a zero has an even multiplicity, then the graph touches the x-axis at that zero, but does not cross it
- If a zero has an odd multiplicity, then the graph crosses the x-axis at that zero in a shape similar to a cubic function

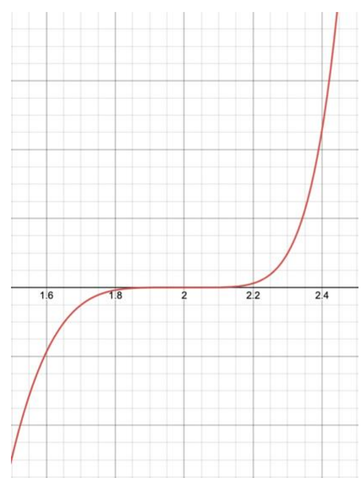
Note! As the multiplicity increases, the graph becomes flatter and flatter around that zero



The factor (x-2) has a multiplicity of 1, so the graph at x = 2 is a straight line



The factor (x-2) has an even multiplicity, so the graph bounces at x = 2 and does not cross the x-axis



The factor (x-2) has an odd multiplicity, so the graph crosses the x-axis at x = 2, and is flatter at x=2 because it has a higher multiplicity

The Intermediate Value Theorem

A theorem that applies to polynomials is the Intermediate Value Theorem. The theorem states that for two numbers a and b in the domain of f, if $a < b$ and $f(a) \neq f(b)$, then the function f takes on every value between $f(a)$ and $f(b)$.
This theorem implies that if a point a on the graph is negative and a point b on the graph is positive, there must be a zero between a and b.

If you struggle with this theorem, think of it in terms of height! If at some point in your life you were 5'2 and you are now 5'8, it means that at some point you were also all the other heights!

Highlight #4: Dividing polynomials

Sometimes it is necessary to divide polynomials. There are two methods to divide polynomials: long division and synthetic division. Long division of polynomials is very similar to decimal long division, and synthetic division is a shortcut for long division. Both are valuable and should be learnt, especially since synthetic division can only be performed when the divisor (the part we are dividing by) has the form x – k, where k is a constant.

Long division

The following steps show how to perform long division

1) Place the numerator (dividend) under the long division symbol $\overline{)\,}$. If the polynomial "skips" any powers write them in with a coefficient of 0.
2) Place the denominator (divisor) to the left of the long division symbol $\overline{)\,}$
3) Divide the first term of the dividend by the first term of the divisor, writing the quotient on top of the long division symbol $\overline{)\,}$
4) Multiply the quotient term by the whole divisor and write it below the dividend
5) Subtract the product from the original dividend (the first term should cancel out)
6) Pull down the next term of the original dividend
7) Repeat steps 3-7 for all following terms of the dividend
8) Divide the remainder by the divisor and add it to the rest of the quotient

Synthetic division

In order to do synthetic division, we only need the coefficients of the dividend as a dividend, and our divisor is the zero k of the divisor x – k. Note! You need to switch the sign of k when using it as a divisor.

Here are the steps to Synthetic division

1) Write k for the divisor

2) Write the coefficients of the dividend (still include the missing terms in the original dividend by putting 0s as the coefficients of the missing terms)
3) Bring the lead coefficient down
4) Multiply the lead coefficient by k. Write the product in the next column
5) Add the terms to the second column
6) Multiply the result by k. Write the result by k. Write the product in the next column.
7) Repeat steps 5-6 for the remaining columns
8) Use the bottom numbers to write the quotient. The number in the last column is the remainder of the division and has degree 0. The next number from the right has degree 0, the next one has degree 1, and so on

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Things you might struggle with

1) Don't confuse the leading term with the leading coefficient
    a. The coefficient is the constant in front of the leading term
2) Take things slowly and check your work

Check Your Learning

1. Graph $\boldsymbol{x^6 + x^5 + 2x^3 + 2x^2}$

2. **2.** Use synthetic and long division to divide $x^3 + 3x^2 - 10x - 19$ by $x + 2$

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

Answers to Check Your Learning

1.

There are a lot of steps in this problem, so let's take it one at a time

1) Find the x and y intercepts

   Let's start by factoring the function $\boldsymbol{x^6 + x^5 + 2x^3 + 2x^2}$

   i. There is a common factor of $\boldsymbol{x^2}$ across all terms, so let's factor it out

   $$\boldsymbol{x^2(x^4 + x^3 + 2x + 2)}$$

   ii. Let's try splitting the part in parenthesis in two groups and check if there are common factors there

   $$\boldsymbol{x^2}[x^3(x + 1) + 2(x + 1)]$$

   iii. Let's factor out the greatest common factor in both groups

   $$\boldsymbol{x^2}(x + 1)(x^3 + 2)$$

   iv. This is now the factored equation

   v. The x intercepts occur at

   $$\boldsymbol{x^2} = 0 \rightarrow x = 0$$

   $$x + 1 = 0 \rightarrow x = -1$$

   $x^3 + 2$ = 0 -> x = $\sqrt[3]{-2}$

   vi. The y intercept occurs at (0,0)

2) Check for symmetry

   a. An even function is symmetric around the y axis, so f(x) = f(-x)

   b. An odd function is symmetric around the origin so f(-x) = -f(x)

   i. In order to check for symmetry we plug in – x

   ii. $\boldsymbol{(-x)^6 + (-x)^5 + 2(-x)^3 + 2(-x)^2}$

   iii. $\boldsymbol{x^6 - x^5 - 2x^3 + 2x^2}$

   iv. As we can see f(-x) is not equal to -f(x) or f(x), so this function is neither odd nor even

3) Identify the multiplicity of the zeroes and graph the behavior at the zeroes

   i. $\boldsymbol{x^2}$ has multiplicity 2

      - Which implies that the graph bounces at x = 0, but does not cross the x-axis

   ii. $(x + 1)$ has multiplicity 1

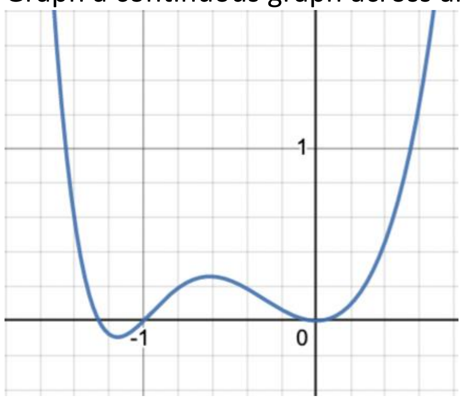
      - Which implies that the graph crosses the x-axis at x = -1 in a straight line)

   iii. $(x^3 + 2)$ has multiplicity 1

      - Which implies that the graph crosses the x-axis at x = $\sqrt[3]{-2}$ in a straight line)

4) Determine the end-behavior by examining the leading term
   i. The leading term is $\boldsymbol{x^6}$**,** which is even
   ii. If we plug in -x into $\boldsymbol{x^6}$ we get $\boldsymbol{x^6}$ (so as x approaches negative infinity, the graph approaches positive infinite)
   iii. If we plug in x into $\boldsymbol{x^6}$ we get $\boldsymbol{x^6}$ (so as x approaches positive infinity, the graph approaches positive infinite)

5) Graph a continuous graph across all the points



6) Ensure the turning points are 1 less than the degree of the polynomial
   The degree of the polynomial is 6, so the number of turning points cannot exceed 6 – 1 = 5. Our graph has 2 local minimums and 1 local maximum, which means there are less than 5 turning points
7) Check the graph with technology (optional)

2.

This problem is also long, so let’s take it one at a time

$$x^3 + 3x^2 - 10x - 19$$

Set up the long division

$$x+2 \;\overline{\big)\, x^3 \quad -3x^2 \quad -10x \quad -19}$$

$$\begin{array}{r} x^2 \qquad\qquad\qquad\quad \\ x+2 \;\overline{\big)\, x^3 \quad -3x^2 \quad -10x \quad -19} \end{array} \qquad \frac{x^3}{x} = x^2$$

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{\big)\, x^3 \quad -3x^2 \quad -10x \quad -19} \\ - & \\ & \underline{x^3 \quad +2x^2 \qquad\qquad\qquad} \\ & \quad -5x^2 \quad -10x \quad -19 \end{array} \qquad \begin{array}{l} \frac{x^3}{x} = x^2 \\ \\ x^2\,(x+2) = x^3 + 2x^2 \end{array}$$

$$
\begin{array}{r|rrrr}
 & x^2 & -5x & & \\
\hline
x+2 & x^3 & -3x^2 & -10x & -19 \\
- & x^3 & +2x^2 & & \\
\hline
 & & -5x^2 & -10x & -19
\end{array}
\qquad \frac{-5x^2}{x} = -5x
$$

$$
\begin{array}{r|rrrr}
 & x^2 & -5x & & \\
\hline
x+2 & x^3 & -3x^2 & -10x & -19 \\
- & x^3 & +2x^2 & & \\
\hline
 & & -5x^2 & -10x & -19 \\
 & & - & & \\
 & & -5x^2 & -10x & \\
\hline
 & & & 0 & -19
\end{array}
\qquad \frac{-5x^2}{x} = -5x \qquad -5x\,(x+2) = -5x^2 - 10x
$$

19 is the remainder.
So we write the solution as

$$\frac{x^3 - 3x^2 - 10x - 19}{x+2} = x^2 - 5x + \frac{-19}{x+2}$$

Long Division Page : https://www.emathhelp.net/calculators/algebra-1/polynomial-long-division-calculator/?numer=x%5E3-3x%5E2-10x-19&denom=x%2B2