Week 7
MTH-1320 – PreCalculus

Hello and Welcome to the weekly resources for MTH-1320 – PreCalculus!

This week is <u>**Week 7 of class**</u>**, and typically in this week of the semester. your professors are covering these topics below.** If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website <u>www.baylor.edu/tutoring</u> or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key Words: Rational functions, asymptotes

Topic of the Week: Rational functions

A rational function is a function that can be written as the quotient of two functions P(x) and Q(x). It is called rational because it is a ratio of two different polynomials. If we let P(x) be a polynomial with degrees n and→ coefficients a, and Q(x) a polynomial with degrees q and coefficients b, then f(x) can be written as

$$f(x) = \frac{P(x)}{Q(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0}{b_q x^q + b_{q-1} x^{q-1} + \ldots + b_2 x^2 + b_1 x + b_0}$$

Highlight #1: Arrow Notation

In order to discuss rational functions, we must first understand arrow behavior. We used arrow notation to discuss end-behavior (what does the function do as x-> ∞), but it can also be used to talk about what a function does as x approaches a certain value a.
Arrow notation is summarized in the following table:

Symbol	Meaning
$x \to a^-$	x approaches a from the left (x < a and very close to a but not equal to a)
$x \to a^+$	x approaches a from the right (x > a and very close to a but not equal to a)

| $x \to \infty$	x approaches infinity (increases with no bound)
$x \to -\infty$	x approaches negative infinity (decreases with no bound)
$f(x) \to \infty$	The output approaches infinity (increases with no bound)
$f(x) \to -\infty$	The output approaches negative infinity (decreases with no bound)
$f(x) \to a$	The output approaches a certain value

Note! The tiny signs next to a indicate the side by which we are approaching a, and they are important f(x) as x approaches a from the right does not have to equal f(x) as x approaches a from the left
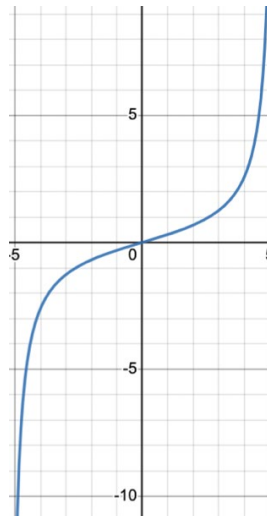
Highlight #2: Asymptotes

An asymptote is a line that a graph approaches but never touches. There are three types of asymptotes: vertical, horizontal and slant asymptotes.
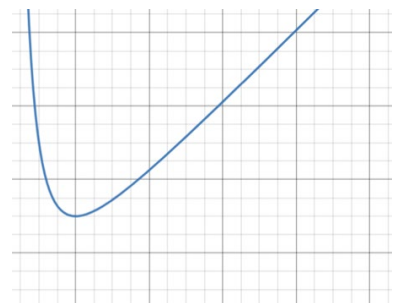The examples below feature an example of each
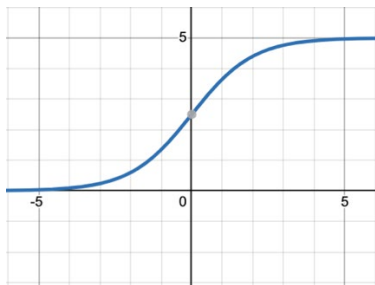
Horizontal asymptote   Vertical asymptote.   Slant asymptote

Finding vertical asymptotes and removable discontinuities

The mathematical definition of a vertical asymptote is "a vertical line where the graph tends towards infinity as the input approaches a".
Note! Vertical asymptotes are caused when the denominator of a rational function equals 0 (as we are dividing by 0, which leads to the output being undefined).
However, be careful. Sometimes, certain outputs that lead to division by 0, do not cause a vertical asymptote, but cause a **removable discontinuity.**

How do we tell the difference between an input that causes a vertical asymptote and one that causes a removable discontinuity?
    1) Factor the numerator and denominator

2) If a factor appears in both the numerator and denominator, there exists a removable discontinuity at that value of x
3) If a factor only appears in the denominator, there exists a vertical asymptote at that value of x

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Finding horizontal and slant asymptotes

The mathematical definition of a horizontal asymptote is a "horizontal line y = b where the graph approaches the line as inputs increase or decrease without bound". Horizontal and slant asymptotes are equivalent to a function's end behavior and are therefore determined by the leading term.

There are three cases that occur with horizontal and slant asymptotes
1) The degree of the numerator is less than the degree of the denominator: horizontal asymptote at y = 0
2) The degree of the numerator is greater than the degree of the denominator: no horizontal asymptote, slant asymptote
3) The degree of the numerator is equal to the degree of the denominator: horizontal asymptote at y = ratio of leading coefficients

A slant asymptote is defined by a linear equation: in order to find the linear equation use long division or synthetic division. The quotient (without the remainder) is the equation of the line of the slant asymptote.

Domain of rational functions

Because f(x) is a quotient of polynomials, and the domain of polynomials are all real numbers, by definition the domain of a rational function is also all real numbers. However, we exclude any value that causes a vertical asymptote or a removable discontinuity from the domain.

Graphs of Rational functions

Graphing a rational function is similar to graphing a polynomial, but we must also consider the asymptotes, as those are values we cannot approach.

These are steps to graphing rational functions:
1) Find the x and y intercepts
2) Factor the numerator and denominator
3) For factors that appear both in the numerator and the denominator, find the removable discontinuities by setting the factors equal to 0 and solve.
4) For factors in the numerator that do not appear in the denominator, determine where each factor is equal to 0 to find the x-intercepts
5) Find the multiplicity of the x-intercepts to determine the local behavior
6) For factors in the denominator that do not appear in the numerator, note the multiplicities of the zeroes to determine the behavior and sketch the vertical asymptotes

7) Compare the degrees of the numerator to determine any horizontal or slant asymptotes
8) Sketch the graph

To determine the local behavior around a vertical asymptote look at the multiplicity of the corresponding factor in the denominator
- If the factor has an odd multiplicity $f(x) \rightarrow -\infty$ on one side and $f(x) \rightarrow \infty$ on the other side
- If the factor has an even multiplicity $f(x) \rightarrow \infty$ on both sides

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Things you might struggle with:
- Identifying removable and not-removable discontinuities (remember to check whether the factor is in both the top and the bottom)
- Finding the equations of slant asymptotes (practice long division)

Check Your Learning
1. Find the vertical asymptote of the following function: $f(x) = \frac{3}{x^2-9}$

2. Find the horizontal asymptote of the following function: $f(x) = \frac{5x^2+7x+12}{3x^2-2x+5}$

3. Find the slant asymptote of the following function: $\frac{x^3+11x^2+36x+36}{x^2+5x+6}$

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

1.

In order to find the vertical asymptotes, we must start by factoring the bottom and setting it equal to 0

$$x^2 - 9 = (x + 3)(x - 3)$$
$$x + 3 = 0 \rightarrow x = -3$$
$$x - 3 = 0 \rightarrow x = 3$$

There are two vertical asymptotes, one at x = 3, and one at x = -3

2.

In order to find the horizontal asymptotes, we must first look at the leading term.

$$f(x) = \frac{5x^2 + 7x + 12}{3x^2 - 2x + 5}$$

The exponents of the leading terms in the numerator and denominator are the same. Hence there is an horizontal asymptote at x = ratio of coefficients of leading term

$$x = \frac{5}{3}$$

3.

In order to find the slant asymptote, we must do polynomial division: $\frac{x^3 + 11x^2 + 36x + 36}{x^2 + 5x + 6}$

$$
\begin{array}{r}
x \quad +6 \\
x^2 + 5x + 6 \enclose{longdiv}{x^3 \quad +11x^2 \quad +36x \quad +36} \\
- \\
\underline{x^3 \quad +5x^2 \quad +6x} \\
6x^2 \quad +30x \quad +36 \\
- \\
\underline{6x^2 \quad +30x \quad +36} \\
0
\end{array}
$$

Therefore, there is a slant asymptote with the equation x + 6

Long Division Page : https://www.emathhelp.net/calculators/algebra-1/polynomial-long-division-calculator/?numer=x%5E3-3x%5E2-10x-19&denom=x%2B2