## Week 8 <br> MTH-1320 - PreCalculus

## Hello and Welcome to the weekly resources for MTH-1320 PreCalculus!

This week is Week 8 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

Key words: Exponential functions, graphs of exponential functions

## Topic of the Week: Exponential Functions

Exponential Functions model outputs that change at a rate proportional to the current quantity, and the general form is

$$
f(x)=a b^{x}
$$

Where $a$ is the initial value $(x=0)$ and $b$ is the base or the growth factor. Both $a$ and $b$ have restrictions:

- a cannot equal 0 (otherwise the term cancels out and $f(x)=0$ )
- b must be positive (else the output oscillates)
- b cannot equal 1 (otherwise $f(x)=a$, since 1 to any power is still 1 )
- $\quad a$ and $b$ must be real numbers

Things to note about exponential functions:

- Domain: $(-\infty, \infty)$
- Range:
- $(0, \infty)$ if a is positive
- $(-\infty, 0)$ if a is negative
- Y -intercept at point ( $0, \mathrm{a}$ )
- There is an horizontal asymptote at $\mathrm{y}=0$;

Note! Remember order of operations when dealing with exponential functions: first raise $b$ to the $x$, and then multiply by a.

What is an exponential function? An exponential function is a function that increases by a certain constant amount (the base) as many times as required by the exponent. A function such as $4^{x}$ quadruples every time $x$ increases by 1 .

| $x$ | $\mathrm{f}(\mathrm{x})=4^{x}$ |  |
| :--- | :--- | :--- |
| 0 | 1 | Relationship with previous output |
| 1 | 4 | $1 * 4$ |
| 2 | 16 | $4^{*} 4$ |
| 3 | 64 | $16^{*} 4$ |

## Highlight \#1: Writing equations with exponential functions

Just like linear equations, based on the information given in the prompt, you can write exponential equations in different ways

- If $a$ and $b$ are given, you can simply substitute them in.

Note! Often prompts say that the output increases by a certain percentage p. You do NOT plug $p$ in for $b$. You must plug $1+p$ into $b$. So, if there is a $25 \%$ increase, $b=1+.25=1.25$

- If you are given two points $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right)$, plug each point in $f(x)=a b^{x}$ and obtain two different equations. Set the two equations equal to each other and find a and b
- If you are given $a$ and $b$ or a point, you can us a blend of the previous methods.
- If you are given a graph, you can identify two points and then use the second method mentioned


## Highlight \#2: The number e and the natural exponential

There is a base that is very often used in mathematics. It is denoted as e, and it represents the number 2.72.
The mathematical definition of e is: as $\mathrm{n}->\infty,\left(1+\frac{1}{n}\right)^{n}->\mathrm{e}$
The function $e^{x}$ is called the natural exponential.
Note! Although it may look like a variable, e is just a constant!

## Highlight \#3: Formulas involving Exponentials

- Compound Interest:

$$
\mathrm{A}(\mathrm{t})=\mathrm{P}\left(1+\frac{r}{n}\right)^{n t}
$$

where

- $A(t)$ is the account value
- t is time measured in years
- P is the starting amount of the account (also called the principal, or present value)
- $r$ is the annual percentage rate (APR) expressed as a decimal
- n is the number of compounding periods in a year
- Continuous Compounding Formula:

$$
\mathrm{A}(\mathrm{t})=\mathrm{P} e^{r t}
$$

where $A(t), P, r$, and $t$ are as defined as in the compound interest

- Continuous growth (or decay) models also use e as a base.

$$
\mathrm{A}(\mathrm{t})=\mathrm{a} e^{r t}
$$

where

- $a$ is the initial value
- $r$ is the continuous growth rate per unit time
- $t$ is the elapsed time


## Highlight \#4: graphs of exponential functions

Graphs of exponential functions with $b>0$ usually look like this

$\mathrm{f}(\mathrm{x})=b^{x}$
b>1


$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=b^{x} \\
& 0<\mathrm{b}<1
\end{aligned}
$$

To graph an exponential function:

1) evaluate the function at three different $x$ values. Make sure to include $x=0$
2) Plot the three points on a coordinate plane
3) Unite the points with a smooth line

Transformations on exponential functions are similar to transformations of any other functions.
Note! Vertical shifts ( $\mathrm{f}(\mathrm{x})=b^{x}+\mathrm{c}$ ) shift the horizontal asymptote from $\mathrm{y}=0$ to $\mathrm{y}=\mathrm{c}$

## Things you might struggle with:

- An exponent just indicates how many times a number is multiplied by itself
- Any exponent less than 1 will decrease the value of the number
- Any exponent more than 1 will increase the value of the number


## Check Your Learning

1. 

Let $\mathrm{f}(\mathrm{x})=2^{x}$. Evaluate
a) $f(2)$
b) $f(4)$
c) $f(5)$
2.

Let $\mathrm{f}(\mathrm{x})=\frac{1}{5}^{x}$. Evaluate
a) $f(2)$
b) $f(3)$
c) $f(4)$

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

## Answers to Check Your Learning

1. 

a) $f(2)=2^{2}=2 * 2=4$
b) $f(4)=2^{4}=2 * 2 * 2 * 2=16$
c) $f(5)=2^{5}=2 * 2 * 2 * 2 * 2=32$
2.
a) $f(2)=\frac{1}{5}^{2}=\frac{1}{5} * \frac{1}{5}=\frac{1}{25}$
b) $f(3)=\frac{1}{5}^{3}=\frac{1}{5} * \frac{1}{5} * \frac{1}{5}=\frac{1}{125}$
c) $f(4)=\frac{1^{4}}{5}=\frac{1}{5} * \frac{1}{5} * \frac{1}{5} * \frac{1}{5}=\frac{1}{625}$

