Week 13
MTH-1321 – Calculus 1

Hello and Welcome to the weekly resources for MTH-1321 – Calculus 1!

This week is <u>Week 13 of class</u>, and typically in this week of the semester your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to **look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester**.

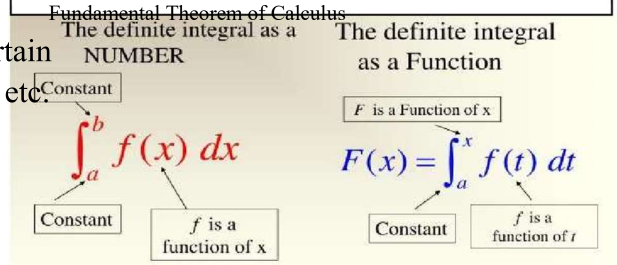
If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

<u>Keywords:</u> Fundamental Theorem of Calculus (2^{nd} part), Distance, Displacement, U-substitution.

<u>Key</u>

- Yellow Highlighting: Definitions that you need to know. Green Highlighting: Explanation of how you actually go about doing the problems.
- Blue Highlighting: Helpful insight into why certain definitions work, how to think about problems, etc.

Another way to represent the 2^{nd} part of the Fundamental Theorem of Calculus



Source: https://www.slideserve.com/rue/average-value-of-a-function-and-the-second-fundamentalcalculus.

<u>Concepts</u>

Chapter 5.5

- In chapter 5.5 the students are learning the 2^{nd} part of the fundamental theorem of calculus. The second part of the fundamental theorem of calculus states that $\frac{d}{dx}\int_a^x f(t)dt = f(x)$. Also, as an implication, if $G(x) = A\,g(x)) = \int_a^{g(x)} f(t)dt$, then $G(x) = A\,(g(x)) * (g'(x)) = g'(x)f(x)$

- Video Resource o https://www.youtube.com/watch?v=q_PNdqlvfPw

- Example Problems o What is the derivate of $G(x) = \int \quad e\ dt$

  - $G(x) = A(x\ ), where\ A(x) = \int \quad e\ dt$, so

    $G\ (x) = A\ (x\ ) * (3x\ )$. Since $A(x) = \int e\ dt$, $A\ (x\ ) = e$. Thus, $G\ (x) = e *$ $(3x\ )$.

  o What is the derivate of $G(x) = \int^{\sqrt{}} tan(t)\ dt$

  - $G(x) = A(\sqrt{x}), where\ A(x) = \int tan(t)dt$, so

    $G\ (x) = A'(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$. Since $A(x) = \int tan(t)dt$,

    $A\ (\sqrt{x}) = tan(\sqrt{x})$. Thus, $G\ (x) = tan(\sqrt{x}) * \left(\frac{1}{2\sqrt{x}}\right)$.

  o What is the derivate of $G(x) = \int \quad ^{()}(t \quad - 5t)\ dt$

  - $G(x) = A(sec(x)), where\ A(x) = \int (t \quad - 5t)dt$, so

    $G\ (x) = A\ (sec(x)) * (sec(x)tan(x))$.

    Since $A(x) = \int (t \quad - 5t)dt$, $A\ (sec(x)) = (sec(x)) \quad - 5sec(x)$.

    Thus, $G\ (x) = (sec(x)) - 5sec(x) * (sec(x)tan(x))$.

  $()$ o What is the derivate of $G(x) = \int (sin(t)cos(t))\ dt$

  - $G(x) = A(ln(x)), where\ A(x) = \int sin(t)cos(t)dt$, so

    $G\ (x) = A \quad ln(x) \quad * \quad \_$.

    Since $A(x) = \int sin(t)cos(t)dt$,

    $A\ (ln(x)) = sin(ln(x))cos(ln(x))$.

    Thus, $G\ (x) = sin(ln(x))cos(ln(x)) * \quad \_$.

  o What is the derivate of $G(x) = \int \quad (e + 3t)\ dt$

  - $G(x) = A(5\ ), where\ A(x) = \int (e + 3t)dt$, so

    $G\ (x) = A\ (5\ ) * (5 \quad * ln(5))$.

    Since $A(x) = \int (e + 3t)dt$,

    $A\ (ln(x)) = (e \quad + 3(5\ ))$.

    Thus, $G\ (x) = (e \quad + 3(5\ )) * (5 \quad * ln(5))$.

Chapter 5.6

- In chapter 5.6 the classes go over one of the uses that exists for an integral. If you need to calculate the displacement of an object over a period of time, and you know the equation for its rate of change you can find the total displacement from $t\ to\ t$ as equal to the integral from $t\ to\ t$ of the rate of change, or in in mathematics
  - $(position\ at\ t = 1) - (position\ at\ t \quad = 2) = \int_{t_1}^{t_2} v(t)dt$
- If you want the total distance traveled (rather than just the change in position), you

  To actually find the absolute value of the function $v(t)$ you need to create a

  need to take the integral of $\int \quad |v(t)|dt$
  - chart like with the 1st derivative test, determine where the function is negative and positive, then split it up into separate integrals. Multiply the integrals over the negative section(s) of the interval to make sure that you get a positive value for each sub-integral.
- Video Resource o https://www.youtube.com/watch?v=Gim1ScMnGCE 1
- Example Problem o What was the total displacement of a person moving at velocity = t from t=1 to t=3? What is the distance traveled by the person?
  - Displacement = $\int t\ dt = 9 - 1 = 8$
  - Distance = $\int |t\ |dt = \int t\ dt = 9 - 1 = 8$
- Since the function t is always non-negative on the interval [1,3], the displacement and the distance will actually be the same.
  - What was the total displacement of a person moving at velocity = $t - 9t$ from t=1 to t=5? What was the distance?
    - Displacement = $\int (t \quad - 9t)dt = (156.25 - 112.5) - (0.25 - 4.5) = 48$
    - Distance = $\int |(t \quad - 9t)|dt = -\int (t \quad - 9t)dt + \int (t \quad - 9t)dt = -((20.25 - 40.5) - (0.25 - 4.5)) + ((156.25 - 112.5) - (20.25 - 40.5)) = 80$
- The function $t - 9t$ is negative from [1,3], so I had to split the integral into two pieces and manually ensure that both pieces would be positive.
  - Follow up question: in the above example we got a larger distance than displacement. Does this make sense? Is there any time that displacement would be greater than distance?
- A: It does make sense, and distance is always greater than or equal to displacement.
  - What was the total displacement of a person moving at velocity = $t - 4t$ from t=0 to t=5? What was the distance?
    - Displacement $= \int_1^5 (t^2 - 4t)dt = (\frac{5^3}{3} - \frac{4(5^2)}{2}) - (\frac{1^3}{3} - \frac{4(1^2)}{2}) = -6.67$

- Does it make sense that we would have a negative displacement?
  - o Yes – displacement can be negative. It is distance that is always positive.
- Distance = $\int |(t \quad - 4t)|dt = -\int (t \quad - 4t)dt + \int (t \quad - 4t)dt =$

$$-\left(\left(\frac{4^3}{3} - \frac{4(4^2)}{2}\right) - \left(\frac{1^3}{3} - \frac{4(1^2)}{2}\right)\right) + \left(\left(\frac{5^3}{3} - \frac{4(5^2)}{2}\right) - \left(\frac{4^3}{3} - \frac{4(4^2)}{2}\right)\right) = 11.33$$

Chapter 5.7

Another way to write U-substitution

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$

where $u = g(x)$ $du = g'(x)$

Source: https://calcworkshop.com/integrals/u-substitution/

- In chapter 5.7 we are introduced to U substitution as a method for solving integrals. U-sub works by replacing the complicated part of an integral with a variable "u," which we use to make the problem much simpler. Mathematically, we are changing the integral from being $\int f\ u(x)\ u'(x)dx$ to looking like $\int f(u)du$. If our integral is indefinite, we will need to switch back from being in terms of "u" at the end of our problem. On the other hand, if our integral is definite, then we will also need to change the bounds of integration from being "a" and "b" to being "u(a)" and "u(b)."
- New Video Resources o https://www.youtube.com/watch?v=-NZU-0j6FJ0 o https://www.youtube.com/watch?v=ipOVrYi_LTE
- Example Problems
  - o 1. $\int x \sin x\ dx$

  - o 2. $\int x\ e^x dx$

Things Students Tend to Struggle With

- Using the 2nd part of the Fundamental theorem of Calculus.
  - Actually applying the 2nd part of the FTC in problems can be really difficult. The problems that require the 2nd part of the FTC tend to be longer and more complicated than your average integration problem. As with all multi-part calculus problems, I recommend trying to break the problem down into its component pieces (in this case A(g(x)) and g(x)), then trying to solve the problem piece by piece, rather than just jumping straight to the final answer.
- Using the U-Substitution method.
  - The key to using U-Sub is being able to determine what your "u" is going to be. For the most part this should be relatively straightforward – it will be the "inner" part of a nested function. Also, if you see that one term of the problem is the derivative of another term, the second term will often be the "u" you need to use.

Thanks for checking out these weekly resources!
Don't forget to check out our website for group tutoring times, video tutorials and lots of other resources: www.baylor.edu/tutoring ! Answers to check your learning questions are below!

ANSWERS to check your learning section

1. $\sin(x) - x\cos(x) + C$
2. $xe^x - e^x + C$