Week - 3
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 3 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135.

The first few weeks of class have been focused on Chapter 12 so we will be covering chapters 12.1 – 12.4 and 12.7 involving Vectors & the Geometry of Space in this resource.

Key Words: Vectors, Dot Product, Cross Product, Cylindrical Coordinates, Spherical Coordinates

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TOPIC OF THE WEEK

Vectors!

Vector: line with both direction and magnitude (real life example: velocity)

- In a plane, vectors are determined by 2 points (initial and terminal)
- In a space, vectors are determined by 3 points (x,y,z)
- Notation: $\overrightarrow{PQ}$ or **PQ**

  $\overrightarrow{PQ} = \langle x, y \rangle = \langle (x_2 - x_1), (y_2 - y_1) \rangle$

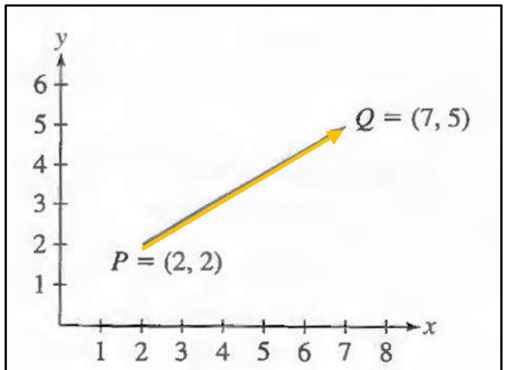
Magnitude: length of a vector

$$\|\overrightarrow{PQ}\| = \sqrt{x^2 + y^2}$$

Ex. (using picture on the right)

$$\overrightarrow{PQ} = \langle (7-2), (5-2) \rangle = \langle 5,3 \rangle$$

$$\|\overrightarrow{PQ}\| = \sqrt{(5)^2 + (3)^2} = \sqrt{34}$$



(A) The vector $\overrightarrow{PQ}$

*Follow the same process for a 3D vector with 3 components, just extend the equations to include z, by adding $(z_2 - z_1)$ or $+ z^2$

Vector Math

Adding Vectors: add their components together

$$\vec{V_1} + \vec{V_2} = \langle x_1, y_1 \rangle + \langle x_2, y_2 \rangle = \langle (x_1 + x_2), (y_1 + y_2) \rangle$$

Scaler Multiplication: multiply each component by scalar, b

$$b * \vec{V} = b\langle x, y \rangle = \langle bx, by \rangle$$

Unit Vector: vector with a length of 1, used to represent direction rather than magnitude

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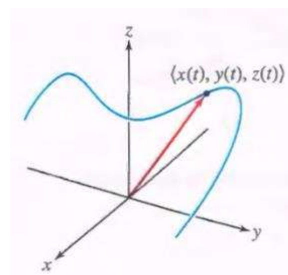
HIGHLIGHT #1: 3 DIMENSIONAL SPACES

Lines / Curves

Distance Formula: Finding the distance between two 3D points on a line

$$|P - Q| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

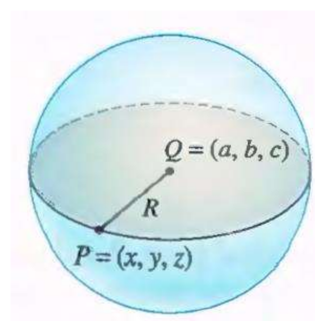
*Notice this is the same formula as magnitude!



Curves in a 3D space must be described parametrically. We describe a point on a curve using the point itself (blue) and a direction vector (red) that is parallel to the curve. We can use vector parameterization and parametric equations.

Vector Parameterization

$$r(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$



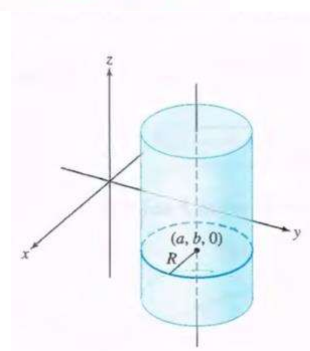
Parametric Equations

$$x(t) = x_0 + at \qquad y(t) = y_0 + bt \qquad z(t) = z_0 + ct$$

Spheres/Cylinders

Equation for a Sphere: where (a, b, c) are the coordinates for the center point of the sphere, R is the radius of the circle

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$$



Equation for a Cylinder: where the cylinder runs parallel to the z-axis

$$(x - a)^2 + (y - b)^2 = R^2$$

*Equations are typically given constraints which further define the surface. Notice the z-axis is not defined in the provided equation for a cylinder. It is assumed, therefore, the surface runs parallel to the z-axis in the +∞ and -∞ directions. Whichever variable is left out, will be the axis that the cylinder is parallel to.

*Notice how similar these equations are. Being able to see and differentiate an equation and then visualize the 3D surface is an integral skill to have moving forward in Calculus 3.

HIGHLIGHT #2: DOT AND CROSS PRODUCT

These equations are both very important vector operations. The applications of these products are used widely in engineering, physics, mathematics, and any situation involving vectors or forces. Therefore, while the equations are important, understanding the answer is essential.

Dot Product:
The dot product of two vectors $a \cdot b$ is a scalar.

$$a = \langle a_1, a_2, a_3 \rangle \qquad b = \langle b_1, b_2, b_3 \rangle$$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Essentially the dot product involves multiplying the corresponding components and adding their products. Hopefully the colors will help visualize this!

The dot product is also related to the angle between the vectors, a and b

$$a \cdot b = \|a\| \, \|b\| \cos\theta$$

The angle between two vectors can be found by rearranging this equation to solve for θ using the magnitudes of both vectors and their dot product. This is an important application of the dot product.

Cross Product:
The cross product of two vectors involves matrices and results in a vector $\langle i, j, k \rangle$.
(i , j , & k are the components of the standard unit vector.) i is the x component; j is the y component; and k is the z component.)

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\boldsymbol{i} - (a_1 b_3 - a_3 b_1)\boldsymbol{j} + (a_1 b_2 - a_2 b_1)\boldsymbol{k}$$

The cross product may look complex but here are a couple tricks to help!

1. Cover up the column you are working on. So for the first component, i, cover the column with your pencil or finger and only focus on the other two columns.
2. Follow the pattern below for the component value

Multiply blue – Multiply orange

$(a_2b_3 - a_3b_2)$

3. Repeat with the j and k columns

*Pay close attention to the signs! The j component is negative!

The following videos are great resources to use for further explaination!

Dot product: https://www.youtube.com/watch?v=86ALeODKd-g
Cross Product: https://www.youtube.com/watch?v=pWbOisq1MJU
Visualization of Both: https://www.youtube.com/watch?v=h0NJK4mEIJU

HIGHLIGHT #3: CYLINDRICAL & SPHERICAL COORDINATES

These coordinate systems, while seeming complicated, are actually meant to make your life less complicated. They are useful for problems with symmetry along an axis or rotational symmetry. The typical coordinate system we use (x, y ,z) is called rectangular.

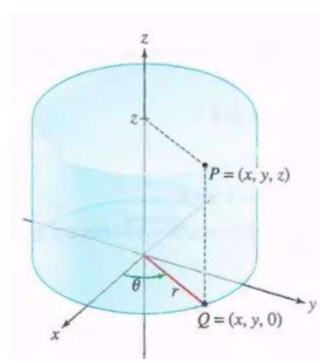
Cylindrical $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{z})$

As the name implies, this set of coordinates is useful for cylinders running along the z-axis.

R and θ are the polar coordinates on the xy plane
z is the height of the cylinder.

Here are the conversion tables:

Cylindrical to rectangular	Rectangular to cylindrical
$x = r\cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\theta$	$\tan\theta = \frac{y}{x}$
$z = z$	$z = z$



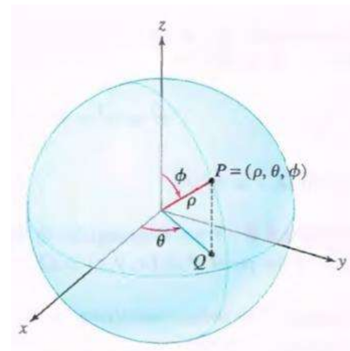
*All angles should be expressed in radians, remember the unit circle!

Spherical (ρ, θ, ϕ)

As this name implies as well, this coordinate system is best for spheres centered at the origin

ρ, rho, represents the radius of the sphere
θ, theta, represents the polar angle on the xy plane
ϕ, phi, is the angle of declination which is the angle in the z direction from the xy plane
The conversions are as below:

Spherical to rectangular	Rectangular to spherical
$x = \rho \sin\phi \cos\theta$	$\rho = \sqrt{x^2 + y^2 + z^2}$
$y = \rho \sin\phi \sin\theta$	$\tan\theta = \dfrac{y}{x}$
$z = \rho \cos\phi$	$\cos\phi = \dfrac{z}{\rho}$



Here is a YouTube video with more examples and explanation!

https://www.youtube.com/watch?v=pmuC2iB8wME

CHECK YOUR LEARNING

1. Find the magnitude of a vector between points $S = (1,3,5)$ and $T = (4,3,6)$.
2. Find the vector parameterization and parametric equations for the line through the point Q = (0,7,2) with the direction vector $\vec{u} = \langle 2,1,7 \rangle$.
3. Find the angle between the vectors in Question #1.
4. Find the cross product between the vectors in Question #1.
5. Convert the point T = $(3\sqrt{3}, -3, 6)$ into cylindrical and spherical coordinates.

THINGS YOU MAY STRUGGLE WITH

1. Understanding each step of the cross product. There are a lot of steps to remember and consider so it's okay to have the formula with you the first few times you do it. With practice you will become more familiar with the signs and which values you multiply vs subtract vs add.

2. It is often difficult knowing when to switch to a different coordinate system. The best way to do this is remembering the names of each and if that's the shape you're aiming for, use it! If you have a space resembling a sphere or part of a sphere use spherical coordinates!

That's all for this week! I hope this was a helpful review of Chapter 12 and the first few weeks of class! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:
1. $\sqrt{10}$
2. $r(t) = \langle 2t, 7+t, 2+7t \rangle$, x(t) = 2t, y(t) = 7+t, z(t) = 2+7t
3. 58.71°
4. 3**i** + 14**j** - 9**k**
5. $(6, \frac{\pi}{6}, 6)$ and $(6\sqrt{2}, \frac{\pi}{6}, \frac{\pi}{4})$