Week 16
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 16 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Final Review

LIST OF IMPORTANT TOPICS
(with referenced resources)

Vectors (Week 3)

- Dot and Cross Product

Different Coordinate Systems (Week 3)

- Rectangular / Polar / Cylindrical / Spherical

Vector Valued Functions (Week 4)

Arc Length (Week 4)

Partial Derivatives (Week 5)

Tangent Planes (Week 5)

Gradients (Week 6)

Directional Derivatives (Week 6)

Optimization (Week 6)

Double Integrals (Week 7)

Triple Integrals (Week 8)

Vector Fields (Week 9)

- Curl and Divergence
- Conservative

Line Integrals (Week 10)

Surface Integrals (Week 11)

Greens Theorem (Week 12)

Stokes Theorem (Week 13)

Divergence Theorem (Week 15)

All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

Important Equations and Operations

Vectors

Magnitude: $\|\overrightarrow{PQ}\| = \sqrt{x^2 + y^2}$ *where* $\overrightarrow{PQ} = \langle x, y \rangle$,

Dot Product: between two vectors, produces a scalar

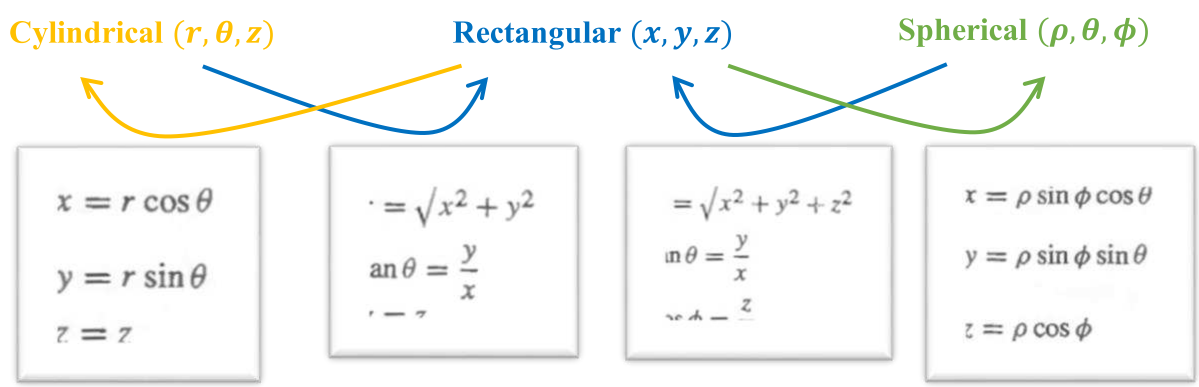
$$\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle \qquad \boldsymbol{b} = \langle b_1, b_2, b_3 \rangle$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Cross Product: between two vectors, produces a vector $\langle i, j, k \rangle$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\boldsymbol{i} - (a_1 b_3 - a_3 b_1)\boldsymbol{j} + (a_1 b_2 - a_2 b_1)\boldsymbol{k}$$

Different Coordinate Systems



Arc Length

$$s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\ dt$$

Gradients & Directional Derivatives

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \qquad D_u f(P) = \nabla f_P \cdot \boldsymbol{u}$$

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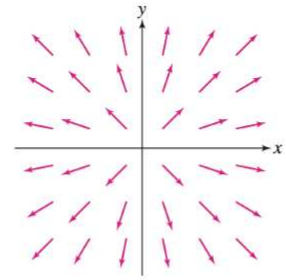
Vector Fields

Divergence: how much the flow is expanding ($div(\boldsymbol{F}) < 0$) or compressing ($div(\boldsymbol{F}) > 0$)



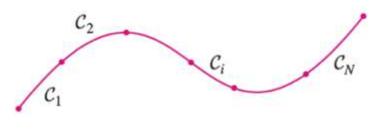
$$div(\boldsymbol{F}) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Curl: how the flow rotates

$$curl(\boldsymbol{F}) = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\boldsymbol{i} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\boldsymbol{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\boldsymbol{k}$$

Line Integrals

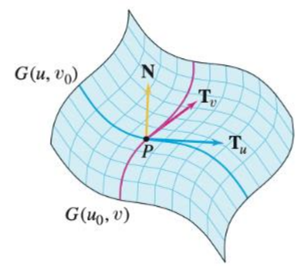
Scalar: find total mass, total charge density, etc.



$$\int_C f(x,y,z)\,ds = \int_a^b f(\boldsymbol{r}(t))\|\boldsymbol{r}'(t)\|\,dt$$

Vector: find work

$$\int_C \boldsymbol{F}\,d\boldsymbol{r} = \int_a^b \boldsymbol{F}(\boldsymbol{r}(t)) \cdot \boldsymbol{r}'(t)\,dt$$



Surface integrals: N is the normal vector

$$\iint_S f(x,y,z)dS = \iint_D f(G(u,v))\|\boldsymbol{N}(u,v)\|du\,dv$$

Important Concepts

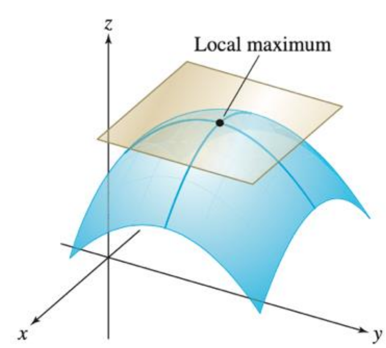
Partial Derivatives

Finding the rate of change with respect to each variable separately

Notation: $f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

The most important key to determining the partial derivatives is to <u>know which variable you are taking the derivative of</u> and <u>treating the other variable(s) as constants</u>.

Optimization



Finding the extreme values of a function

<u>Step 1:</u> Find critical points using partial derivatives

$$f_x(a,b) = 0 \; and \; f_y(a,b) = 0 \; or \; do \; not \; exist$$

<u>Step 2:</u> use second derivative test to determine the type of critical points

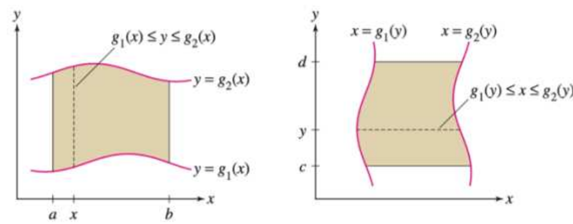
All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

$$D = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}^2(a,b)$$

(i) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.
(ii) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.
(iii) If $D < 0$, then f has a saddle point at (a,b).
(iv) If $D = 0$, the test is inconclusive.

Double / Triple Integrals

Subsequent integration of one variable at a time
Used to find volume of a 3D shape



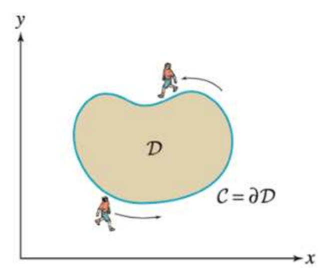
(A) Vertically simple region (B) Horizontally simple region

$$\int_a^b \left(\int_c^d f(x,y)\ dy \right) dx$$

$$\iiint_B f(x,y,z)\, dV = \int_{x=a}^{b} \int_{y=c}^{d} \int_{z=p}^{q} f(x,y,z)\, dz\, dy\, dx$$

Green's Theorem

For non-conservative 2D surfaces in simple closed curves

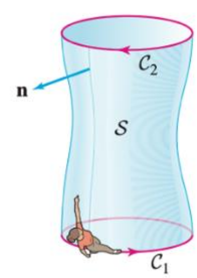


$$\oint_{\partial D} F_1 dx + F_2 dy = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$$

Stokes' Theorem

For non-conservative 3D surfaces that have single closed curves

$$\oint_{\partial S} \boldsymbol{F} \cdot d\boldsymbol{r} = \iint_S curl(\boldsymbol{F}) \cdot d\boldsymbol{S}$$



Divergence Theorem

Solves for flow rate or flux of a closed 3D surface

$$\iiint_{\mathcal{W}} \operatorname{div}(\mathbf{F})\, dV = \iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$$

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CHECK YOUR LEARNING

1. Find the dot and cross product of the vectors $S = \langle 1, 3\ ,5\rangle$ and $T = \langle 4, 3\ 6\rangle$
2. Find the partial derivative in terms of x and then y of $f(x, y) = e^{-x} + \sin\,(x + 2y)$
3. Evaluate $\iiint_W z\, dV$ where W is the region between the planes $z = x + y$ and $z = 3x + 5y$ over the rectangle $D = [0,3] \times [0,2]$.
4. Find the divergence and curl of $\boldsymbol{F} = \langle xy,\ e^x, y + z\rangle$.

THINGS YOU MAY STRUGGLE WITH

1. There is a lot to remember, especially for a comprehensive final exam but hopefully this will help provide a list of the general equations and concepts to know. With these, practice looking at problems, figuring out what you need to find, and detailing the path to get there. Being able to connect what a question asks to what you need to do will help you immensely.
2. For any problem, take extra time and care to set up the problem correctly. Make sure you have the right variables, the right bounds, the right operations, etc. Even if you aren't sure exactly how to solve a problem if you can at least set up the calculus parts of it, you will get more points compared to setting it up incorrectly.

That's all for this semester! I hope this is a helpful review and GOOD LUCK ON YOUR FINALS! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $43, 3\boldsymbol{i} + 14\boldsymbol{j} - 9\boldsymbol{k}$
2. $f_x = -e^{-x} + \cos(x + 2y)\,, f_y = 2\cos\,(x + 2y)$
3. 294
4. $div = y + 1,\ curl =\ \boldsymbol{i} + (e^x - x)\boldsymbol{k}$

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