Week 11
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 11 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Parameterized Surfaces, Surface Integrals, Surface Integrals of Vector Fields

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TOPIC OF THE WEEK

Scalar Surface Integrals

Surface Integrals: integration over a surface of a solid
Two types: scalar and vector surface integrals

Scalar Surface Integrals

- Just like we used parameterized curves for line integrals, we use parameterized surfaces for surface integrals
- Parameterized surfaces have the form:

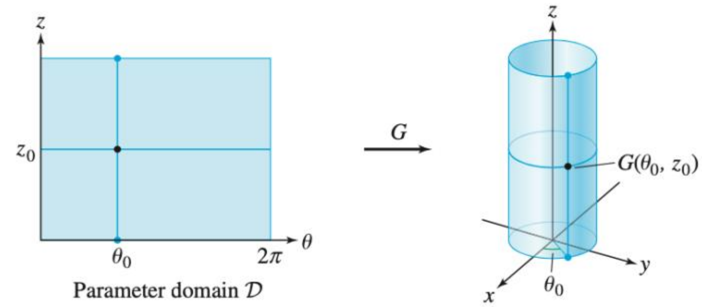
$$G(u,v) = (x(u,v), y(u,v), z(u,v))$$

However, there are more specific parameterization equations of commonly used surfaces:

Parameterization of a Cylinder: a vertical cylinder with radius R

- Parameterized in terms of cylindrical coordinates to wrap rectangle around cylinder

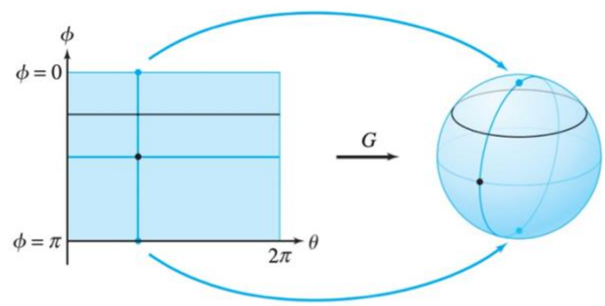
$$G(\theta, z) = (R\cos\theta, R\sin\theta, z), \quad 0 \le \theta \le 2\pi, h - \infty \le z \le \infty$$



Parameterization of a Sphere: with radius R

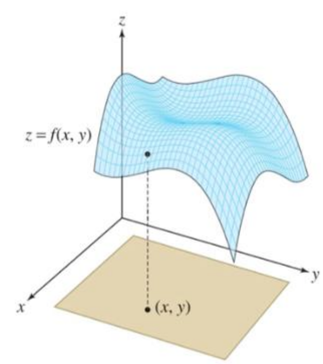
- Parameterized by spherical coordinates to wrap rectangle around the sphere

$$G(\theta, \phi) = (R\cos\theta\sin\phi, R\sin\theta\sin\phi, R\cos\phi), \quad 0 \le \theta \le 2\pi, \ 0 \le \phi \le \pi$$



Parameterization of a Graph: graph of a function $z = f(x, y)$

$$G(x, y) = (x, y, f(x, y))$$



Tangent and Normal Vectors

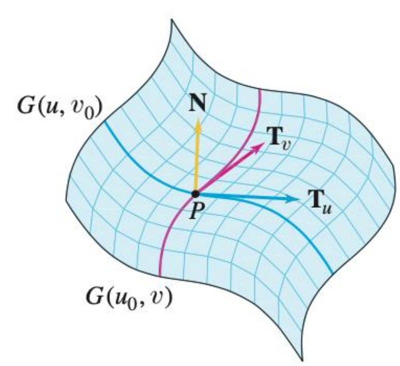
N is the normal vector to the tangent plane while $\mathbf{T}_u$ and $\mathbf{T}_v$ are the tangent vectors.

- If the cross product at the point P is nonzero, then the parameterization G is regular

$$\boldsymbol{N}(P) = \boldsymbol{N}(u_0, v_0) = \boldsymbol{T}_u(P) \times \boldsymbol{T}_v(P)$$

For $G(u, v_0)$: $\quad \mathbf{T}_u(P) = \dfrac{\partial G}{\partial u}(u_0, v_0) = \left\langle \dfrac{\partial x}{\partial u}(u_0, v_0), \dfrac{\partial y}{\partial u}(u_0, v_0), \dfrac{\partial z}{\partial u}(u_0, v_0) \right\rangle$

For $G(u_0, v)$: $\quad \mathbf{T}_v(P) = \dfrac{\partial G}{\partial v}(u_0, v_0) = \left\langle \dfrac{\partial x}{\partial v}(u_0, v_0), \dfrac{\partial y}{\partial v}(u_0, v_0), \dfrac{\partial z}{\partial v}(u_0, v_0) \right\rangle$



The magnitude of the normal vector is defined as: $\|\boldsymbol{N}\| = R^2 sin\phi$

- The $\|\boldsymbol{N}\|$ is considered the <u>distortion factor</u> which measures how the area of a small rectangle is altered under the map G.

Surface Area and Surface Integral

If $G(u, v)$ is a parameterization of a surface S with a domain D assuming G is continuously differentiable, one-to-one, and regular

<u>Surface Integral:</u>

$$\iint_S f(x, y, z) dS = \iint_D f(G(u, v)) \|\boldsymbol{N}(u, v)\| du\, dv$$

<u>Surface Area:</u>

$$Area(S) = \iint_D \|\boldsymbol{N}(u, v)\| du\, dv$$

*When parameterizing a graph parameterized by $G(x, y) = (x, y, g(x, y))$:

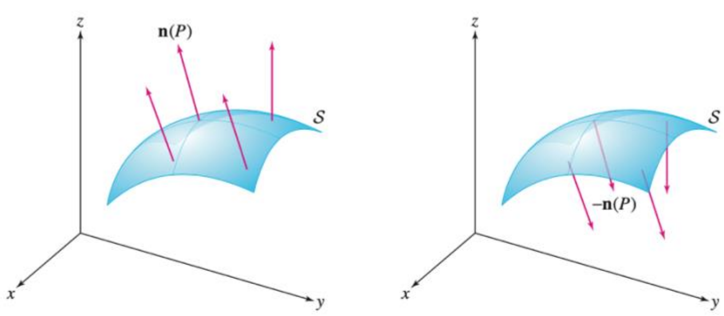
$$\boldsymbol{T}_x = (1, 0, g_x), \qquad \boldsymbol{T}_y = (0, 1, g_y),$$

$$\|\boldsymbol{N}\| = \sqrt{1 + g_x^2 + g_y^2}$$

HIGHLIGHT #1: Surface Integrals of Vector Fields

Surface Integrals of Vector Fields: these integrals represent flux or rates of flow through a surface

- The <u>orientation</u> or direction of the flow must be specified using the unit normal vector $\mathbf{n}(P)$.



(A) One possible orientation of S (B) The opposite orientation

Vector Surface Integral: given the oriented parameterization of $G(u, v)$ over a surface S with a domain D assuming G is continuously differentiable, one-to-one, and regular

$$\iint_S F \cdot dS = \iint_D \boldsymbol{F}(G(u, v)) \cdot \boldsymbol{N}(u, v) du\, dv$$

To find the fluid flux through a surface, just replace **F** with **v** which represents the velocity vector field or flow rate!

Useful videos for further explanation of surface integrals!

Evaluating a Surface Integral
https://www.youtube.com/watch?v=SnUuJ7lRTNw

Surface Integrals (Scalar and Vector)
https://www.youtube.com/watch?v=Gml1HT4y3_c

CHECK YOUR LEARNING

1. Compute $\boldsymbol{T}_\theta, \boldsymbol{T}_z$, and $\boldsymbol{N}(\theta, z)$ for the parameterization $G(\theta, z) = (2\cos\theta, 2\sin\theta, z)$ of the cylinder $x^2 + y^2 = 4$.
2. Calculate the surface area of the portion S of the cone $x^2 + y^2 = z^2$ lying above the disk $x^2 + y^2 \leq 4$.
3. From problem 2, calculate $\iint_S x^2 z\, dS$.

4. Calculate $\iint_S \boldsymbol{F} \cdot\, d\boldsymbol{S}$ where $\boldsymbol{F} = \langle 0,0,x\rangle$ and S is the surface with parameterization $G(u,v) = (u^2, v, u^3 - v^2)$ for $0 \le u \le 1,\ 0 \le v \le 1$ and oriented by upward-pointing normal vectors.

THINGS YOU MAY STRUGGLE WITH

1. One of the biggest challenges of Calculus III is visualizing surfaces. If you can visualize the surface, you are taking an integral of it may be easier to avoid making mistakes since you can visualize bounds and determine if your answer is reasonable.

2. For surface integrals with vector fields, ensure there is an understanding of the direction / orientation of the integration. If the direction is negative, the integration needs to be negative.

That's all for this week! I hope this was a helpful review of 16.4 and 16.5! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $\boldsymbol{T}_\theta = \langle -2\sin\theta\,, 2\cos\theta\,, 0\rangle,\ \ \boldsymbol{T}_z = \langle 0,0,1\rangle,\ \ \boldsymbol{N}(\theta, z) = 2\cos\theta\,\boldsymbol{i} + 2\sin\theta\,\boldsymbol{j}$
2. SA = $4\sqrt{2}\pi$
3. $\frac{32\sqrt{2}\pi}{5}$
4. 1/2