Week 13
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 13 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Stokes' Theorem, Surface Independence

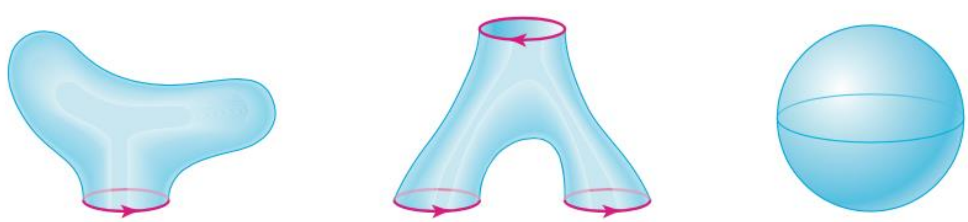
TOPIC OF THE WEEK

Stokes' Theorem

Stokes' Theorem is an extension of Green's Theorem for three dimensional shapes. Therefore, rather than using double integrals, we will now use surface integrals to find the circulation over a surface.

Basic Vocabulary

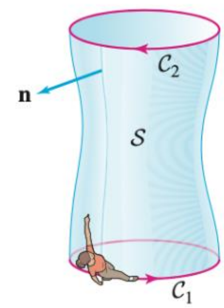
- There are different types of surfaces based on their boundaries



(A) Boundary consists of a single closed curve. (B) Boundary consists of three closed curves. (C) Closed surface (the boundary is empty)

All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

- <u>Boundary orientation:</u> direction as if you were walking along a boundary curve so that the surface is always on your left



Equation

Let S be a surface and **F** be a vector field whose components have continuous partial derivatives on an open region containing S,

$$\oint_{\partial S} \boldsymbol{F} \cdot d\boldsymbol{r} = \iint_S curl(\boldsymbol{F}) \cdot d\boldsymbol{S}$$

If the surface S is a closed surface, then the surface integral is equal to zero!

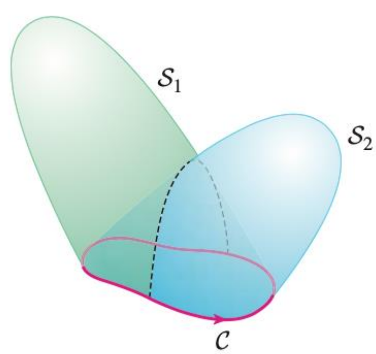
Recall that the curl of a **F** can also be written in this form:

$$\oint_{\partial S} \boldsymbol{F} \cdot d\boldsymbol{r} = \iint_S (\nabla \times \boldsymbol{F}) \cdot d\boldsymbol{S}$$

Recall that the real-world application of vector surface integrals is flux!

HIGHLIGHT #1: Surface Independence

Imagine we have two surfaces, S_1 and S_2 that are defined by the same boundary condition, C.



They would both have a vector field **A** that goes through them. This vector field can also be called the vector potential for **F**.

So, If $\boldsymbol{F} = curl(\boldsymbol{A})$ then the surface integral of **F** through a surface S depends only on the oriented boundary ∂S and not on the surface itself.

$$\iint_S \boldsymbol{F} \cdot d\boldsymbol{S} = \oint_{\partial S} \boldsymbol{A} \cdot d\boldsymbol{r}$$

Useful videos for further explanation of Stokes' Theorem!

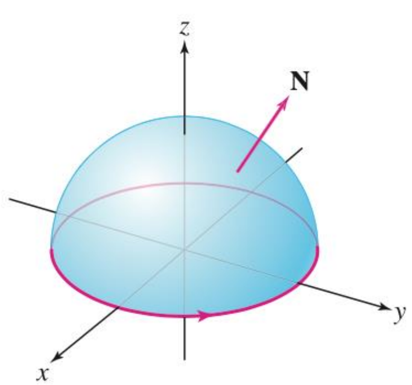
https://www.youtube.com/watch?v=QS-zUSu-nxA

https://www.youtube.com/watch?v=-hqu9sIuAwM&list=RDLVQS-zUSu-nxA&index=3

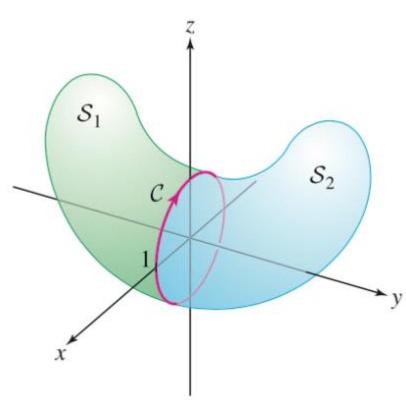
CHECK YOUR LEARNING

1. Use Stokes' Theorem to calculate the circulation of the upper hemisphere with outward pointing normal vectors where $\boldsymbol{F}(x, y, z) = \langle -y, 2x, x + z \rangle$ and $S = \{(x, y, z): x^2 + y^2 + z^2 = 1, z \geq 0\}$.

2. Let $\boldsymbol{F} = curl(\boldsymbol{A})$ where $\boldsymbol{A}(x, y, z) = \langle y + z, \sin(xy), e^{xyz} \rangle$. Find the flux of **F** outward through the surfaces S_1 and S_2 whose common boundary C is the unit circle in the xz-plane.



Problem 1



Problem 2

All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

THINGS YOU MAY STRUGGLE WITH

1. It can be difficult to remember which theorems or equations to use for specific problems. Be sure to take time to learn each theorem or equation and understand its main purpose. Knowing this will help you be able to more clearly see when a specific problem can be aligned with a specific theorem.

2. If you're having difficulty remembering how to perform surface integrals feel free to review the resource for Week 11 and/or review the resource for Week 12 on Green's Theorem.

That's all for this week! I hope this was a helpful review of 17.2! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:
1. 3π
2. $Flux = \pi$

All pictures, tables, and information if property of Calculus Early Transcendentals (4^{th} Edition) by Rogawski, Adams, and Franzosa.