Week 15
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 15 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Divergence Theorem

TOPIC OF THE WEEK

Divergence Theorem

Divergence Theorem is the last fundamental theorem we will cover and relates to both Green's and Stokes' Theorem.

Recall:

Green's Theorem:

$$\underbrace{\iint_{\mathcal{D}} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA}_{\text{Integral of derivative over domain}} = \underbrace{\int_{C} \mathbf{F} \cdot d\mathbf{r}}_{\text{Integral over boundary curve}}$$

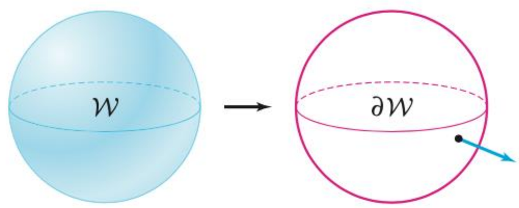
Stokes' Theorem:

$$\underbrace{\iint_{S} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}}_{\text{Integral of derivative over surface}} = \underbrace{\int_{C} \mathbf{F} \cdot d\mathbf{r}}_{\text{Integral over boundary curve}}$$

So, Divergence Theorem will follow the same structure:

$$\underbrace{\iiint_{\mathcal{W}} \operatorname{div}(\mathbf{F})\, dV}_{\text{Integral of derivative over 3D region}} = \underbrace{\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}}_{\text{Integral over boundary surface}}$$

For divergence theorem, S is a closed surface that encloses a three-dimensional region W (also notated as ∂W) sometimes. It is oriented with normal vectors pointing outside of W.



Three-dimensional ball — Boundary is a sphere, oriented to the outside.

Interpretation of Divergence Theorem:

The theorem solves for the flow rate or the flux per unit volume across S.

HIGHLIGHT #1: **Basic Operations on Functions and Vector Fields**

Here is a summary of the basic operations on functions and vector fields.

$$\underset{\text{function}}{f} \xrightarrow{\nabla} \underset{\text{vector field}}{\mathbf{F}} \xrightarrow{\text{curl}} \underset{\text{vector field}}{\mathbf{G}} \xrightarrow{\text{div}} \underset{\text{function}}{g}$$

With this, we can also see that the result of two consecutive operations is zero:

$$\operatorname{curl}(\nabla(f)) = \mathbf{0}, \qquad \operatorname{div}(\operatorname{curl}(\mathbf{F})) = 0$$

Useful videos for further explanation of Divergence Theorem!

https://www.youtube.com/watch?v=vZGvgru4TwE

https://www.youtube.com/watch?v=UOG3mOhv5Xo

CHECK YOUR LEARNING

1. Use the Divergence Theorem to evaluate $\iint_S \langle x^2, z^4, e^z \rangle \, d\boldsymbol{S}$, where S is the boundary of the box W shown in Figure 1.

2. Compute the flux of $\boldsymbol{F} = \langle z^2 + xy^2, \cos(x+z), e^{-y} - zy^2 \rangle$ through the boundary of the surface S in Figure 2.

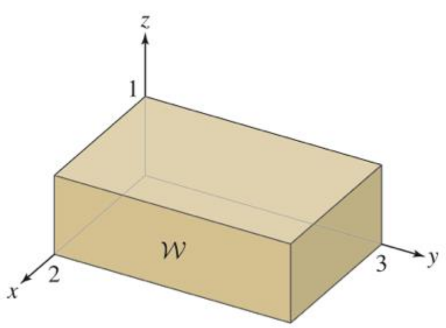


Figure 1

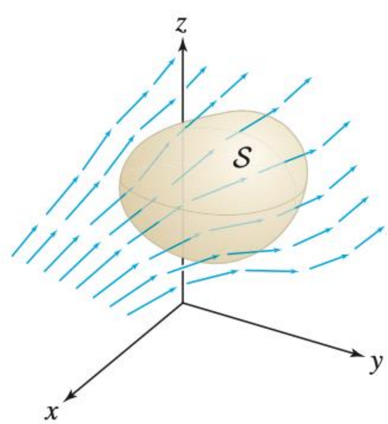


Figure 2

THINGS YOU MAY STRUGGLE WITH

1. The most difficult part about all the fundamental theorems is remembering which ones apply when. Be sure to take time to understand the purpose of each theorem and the equation that go with it since they all apply to different scenarios.

That's all for this week! I hope this was a helpful review of 17.3! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $6e + 6$
2. $Flux = 0$