Week 4
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 4 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

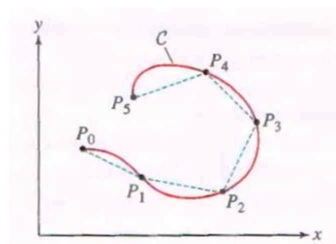
We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

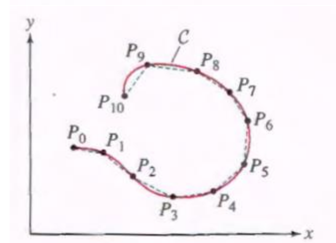
Key Words: Arc Length, Speed, Vector-Valued Functions

TOPIC OF THE WEEK

Arc Length



$N = 5$



$N = 10$

Arc Length: length of a curve

- Typically arc length is defined as: $s = r\theta$ but this equation doesn't correspond well to curves
- Therefore, we will be defining the arc length in parametric form to approximate the length of a curve using a series of straight lines
- As seen from the graphs on the right, the more line segments (N) used to define a curve, the more accurately it represents said curve
    - So, we use integration to make an infinite number of lines!

Arc Length Parameterization Equation: for a curve from points a to b defined by x(t) and y(t):

$$s = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\ dt$$

*In physics, this equation describes the distance traveled along a path from a to b!

The formula for the arc length can be manipulated to calculate different values useful in math and physics.

Speed: since speed is the derivative of position/distance, the speed along a parameterized path is equivalent to the derivative of the equation used for the parameterized arc length

$$speed = \frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2}$$

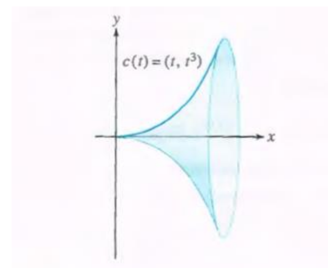
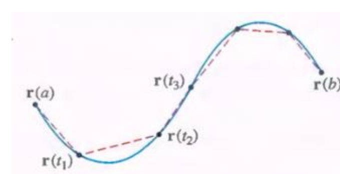


FIGURE 6 Surface generated by revolving the curve about the x-axis.

Surface Area: if a surface is generating by rotating a curve around an axis, its corresponding surface area can be calculated

$$S = 2\pi \int_a^b y(t)\sqrt{x'(t)^2 + y'(t)^2}\ dt$$

Arc Length with Vectors: rather than using straight lines, vectors $\overrightarrow{r(t)}$ can also be used to parameterize curves.



$$s = \int_a^b \|r'(t)\|\ dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\ dt$$

*This equation is just the magnitude of the derivative of the vectors. The equations of speed and surface area can be manipulated to use vectors as well!

HIGHLIGHT #2: VECTOR-VALUED FUNCTIONS

Vector-Valued Functions: functions defined by sets of variables

- These functions are useful in Calculus because they can be differentiated, integrated, and have other properties

Limits: to find the limit of a vector-valued function, one must find the limit of each individual component

$$\lim_{t\to t_0} r(t) = \langle \lim_{t\to t_0} x(t), \lim_{t\to t_0} y(t), \lim_{t\to t_0} z(t)\rangle$$

Derivatives: to take the derivative, the derivative of each component is taken separately

$$r'(t) = \langle x'(t),\ y'(t),\ z'(t)\rangle$$

*The regular rules for derivatives still stand for vector valued functions including, but not limited to: chain rule, dot product and cross product.

Integration: component-based integration is used to find the integral of a vector-valued function

$$\int_a^b r(t)dt = \langle \int_a^b x(t)dt, \int_a^b y(t)dt,\ \int_a^b z(t)dt\rangle$$

The following videos are great resources to use for further explanation of vector-valued functions!

https://www.khanacademy.org/math/ap-calculus-bc/bc-advanced-functions-new/bc-9-4/v/position-vector-valued-functions

https://www.youtube.com/watch?v=Up0ct2rdrFo

CHECK YOUR LEARNING

1. Find the arc length of the curve in parametric form defined by $c(t) = (t^3 + 3, t^2)$ for $0 \le t \le 2$.
2. Find the corresponding surface area for function in problem 1.
3. Find the arc length of **r**(t) = $\langle \sin(4t), \cos(4t), 6t^2\rangle$ for $0 \le t \le 2\pi$.
4. Find the integral of **r**(t).

THINGS YOU MAY STRUGGLE WITH

1. It may be difficult understanding which equation to use when. This comes down to the details. Pay close attention to notation and context clues to determine whether you are given a regular function or a vector valued function. Recall, vectors have the pointed brackets <> while regular functions have curved brackets (). Also, vectors are either bolded or have the vector "hat", $\vec{r}$.

2. This is the time to start re-remembering your Calculus 1 skills involving limits, derivatives, and integrals. Knowing your trig functions, unit circle, and other special derivatives/integrals while not mandatory, will help you solve problems faster.

That's all for this week! I hope this was a helpful review of Chapters 11 and 13! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. 9.073
2. 131.157
3. $8\pi\sqrt{2}$
4. 496.1