# Week 5
# MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 5 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

***Key Words***: Partial Derivatives, Tangent Planes, Differentiability

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## TOPIC OF THE WEEK

## Partial Derivatives

**Partial Derivatives:** rates of change with respect to each variable separately

- A function $f(x, y)$ therefore has two partial derivatives: $f_x$ and $f_y$
- Notation: $\frac{\partial f}{\partial x}$ ,this symbol is essentially a "rounded d" and is sometimes called "del"

The most important key to determining the partial derivatives is to know which variable you are taking the derivative of and treating the other variable(s) as constants.

**Ex**. Find the partial derivatives of $f(x, y) = 4x^3y^2 + 3x + e^y$

Step 1: Find the partial in terms of x

$$f_x(x, y) = \frac{\partial}{\partial x}(4x^3y^2 + 3x + e^y)$$

Step 2: Treat the variable y as a constant

$$\frac{\partial}{\partial x}(4x^3y^2 + 3x + e^y)$$

Treat $y^2$ as a constant so only the derivative of $4x^3$ is taken and $y^2$ is left alone. (pretend $y^2 = 2$)

$$\frac{d}{dx}(4x^3 * 2)$$

$$= 12x^2 * 2$$

$$= 12x^2 * y^2$$

Since there is no y in this term, just take the derivative like usual

$$\frac{d}{dx}(3x)$$

$$= 3$$

Since there is no x in this term, the whole thing is treated as constant (pretend $e^y = 7$)

$$\frac{d}{dx}(7)$$

$$= 0$$

$$f_x(x, y) = 12x^2y^2 + 3 + 0$$

Step 3: Find the partial derivative in terms of y

$$f_y(x, y) = \frac{\partial}{\partial y}(4x^3y^2 + 3x + e^y)$$

Step 4: Treat the variable x as a constant, using the same thinking as Step 2

$$f_y(x, y) = 4x^3 * 2y + 0 + e^y$$

$$f_y(x, y) = 8x^3y + e^y$$

*All derivative rules (chain, product, quotient) still apply in the same way if the variable not being solved for is treated as a constant!

## HIGHLIGHT #1: HIGHER ORDER PARTIAL DERIVATIVES

These are similar to regular higher order derivatives, except you must pay attention to which variable you are deriving for, for each subsequent derivation.

There are two types of higher order partials: second-order and mixed partials

**Second order:** taking the derivative of the same variable twice

- Notation: $f_{xx}$ or $f_{yy}$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

**Mixed partials:** taking the first derivative in terms of one variable then taking the second derivative in terms of a different variable

- Notation: $f_{xy}$ or $f_{yx}$
- Be careful about the order! If it is written $f_{xy}$ then it goes left to right, if written $\frac{\partial f}{\partial y \partial x}$ then it goes right to left. Both ask to take the derivative of x, then y.

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

- Clairaut's Theorem states that if $f_{xy}$ and $f_{yx}$ both exist and are continuous then:

$$f_{xy} = f_{yx}$$

And this is true for any higher order differential, so:

$$f_{xxyy} = f_{xyxy} = f_{yxxy} = f_{xyyx} = f_{yyxx} = f_{yxyx}$$

This works because no matter what, we are differentiating x twice, and y twice so the order doesn't matter

Because of this principle, **we have the freedom to choose the order** which we differentiate so choose wisely to make the problem easiest!

Ex. Calculate partial derivative of $g_{zzxy}$ where $g(x, y, z) = \arcsin\left(\frac{y^2 z}{e^z}\right) + x^3 y^2 z$

If we choose to take the partial in terms of x first, the ugly arcsin term immediately goes away since there is no x term in it which significantly simplifies the rest of the problem! Do $g_{xzzy}$ instead!

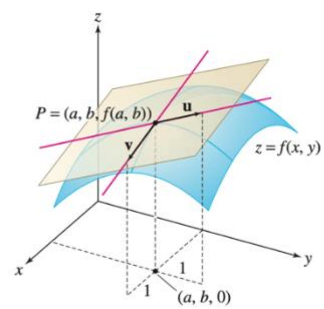
Useful videos to further explain partial derivatives:

https://youtu.be/SbfRDBmyAMI
https://mathinsight.org/partial_derivative_examples
https://www.youtube.com/watch?v=BUIleGfqAeo

## HIGHLIGHT #2: TANGENT PLANES & DIFFERENTIABILITY

**Tangent Plane:** tangent lines are for single variable equations $f(x)$, and tangent planes are for functions with two variables $f(x, y)$

- Need a point (P) on the plane and a normal vector to define the plane
- The normal vector is the cross product of the two tangent vectors (u and v) at the point on the plane




How do we find the tangent vectors?

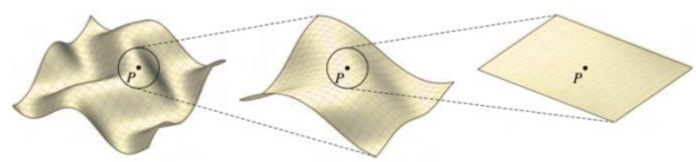
- We can find one tangent line in the x direction and one in the y direction and find their slopes using the partial derivatives

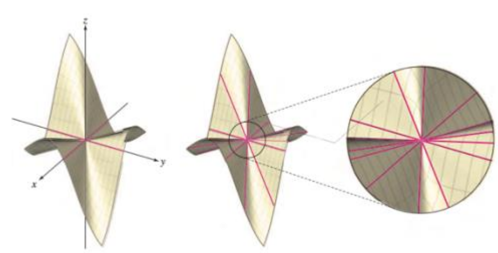
**General equation** for the tangent plane if f(x,y) is locally linear: where a and b are the x and y coordinates of the point P

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

**Locally linear:** A plane is locally linear if as we zoom in on the graph at point P, the graph looks flatter

- Observe the pictures on the right. The top picture is locally linear while the bottom is not.




**Differentiability:** if $f_x(x,y)$ and $f_y(x,y)$ exist and are continuous, then f (x,y) is differentiable on D.

- Also it is differentiable at (a,b), if it is locally linear

<u>General Problem-Solving Steps:</u>

1. Find the partial derivatives and determine if they exist and are continuous for all (x,y)
   a. If so, then they can be considered locally linear and differentiable
2. Fill in the general equation for a tangent plane

The following videos are great resources to use for further explanation of tangent planes!

https://www.youtube.com/watch?v=pxmW8_Cpd7U

https://www.khanacademy.org/math/multivariable-calculus/applications-of-multivariable-derivatives/tangent-planes-and-local-linearization/v/computing-a-tangent-plane

All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

## CHECK YOUR LEARNING

1. Find the partial derivative in terms of x of $f(x, y) = e^{-x} + \sin(x + 2y)$
2. Find the partial derivative in terms of y of the equation in #1.
3. Determine $g_{xxy}$ for $g(x, y) = e^{xy} + 3y$
4. Determine $g_{yxx}$ for the equation in #3.
5. Find the tangent plane of the graph of $f(x, y) = xy^3 + x^2$ at (1,3)

## THINGS YOU MAY STRUGGLE WITH

1. Partial derivatives are difficult at first because it's hard to imagine a variable as a constant. This will get easier with lots of practice and if you take your time and really think about what you need to do for each term. If it helps to rewrite the variables you are treating as constants as actually values, do it!

2. Some of the terms for tangent planes may sound difficult but it's nothing you haven't done before. Look for where the partials may be undefined, if there is nothing, then it is locally linear and differentiable, and you can proceed as normal with the general equation.

That's all for this week! I hope this was a helpful review of Chapter 14.3 and 14.4! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

**Answers:**

1. $f_x = -e^{-x} + \cos(x + 2y)$
2. $f_x = 2\cos(x + 2y)$
3. $g_{xxy} = xy^2e^{xy} + 2ye^{xy}$
4. $g_{yxx} = xy^2e^{xy} + 2ye^{xy}$
5. z = 29x+27y-82