Week 7
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 7 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Double Integrals, Iterated Integrals, Horizontally/Vertically Simple

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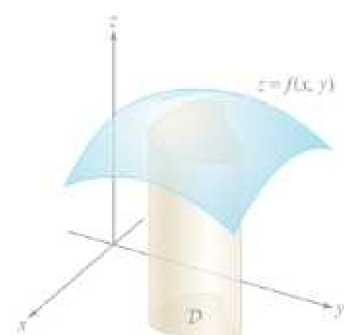
TOPIC OF THE WEEK

Double Integrals

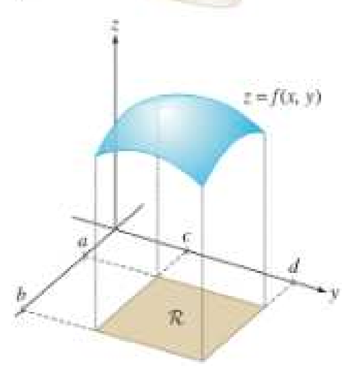


Double integral: Integration by two variables: x and y

- Represents the volume of a solid region between the graph of $f(x, y)$ and the domain D in the x-y plane

$$\iint_D f(x, y)\, dA$$

- The most common method of solving double integrals is using the Fundamental Theorem of Calculus (FTC), just like single integrals



Iterated Integrals: form needed to use the FTC where the domain is represented by a rectangle R which defines the bounds of the area $R = [a, b]$ x $[c, d]$

$$\int_a^b \left(\int_c^d f(x, y)\; dy \right) dx$$

- The limits a and b are for the variable x, while c and d are for the variable y
- Therefore, we integrate by y first and then x

Ex. Find the volume between the graph $f(x, y) = ye^x$ and the rectangle $R = [2,4] \times [1,9]$

This can be rewritten as the integrated integral:

$$\int_2^4 (\int_1^9 ye^x \, dy) \, dx$$

Step 1: Evaluate the inner integral with respect to y, treat x as a constant

$$\int_1^9 ye^x \, dy = e^x \int_1^9 y \, dy$$

$$e^x \left[\frac{1}{2}y^2\right]_1^9$$

$$\frac{e^x}{2}(81 - 1) = 40e^x$$

Step 2: Evaluate the outer integral with respect to x

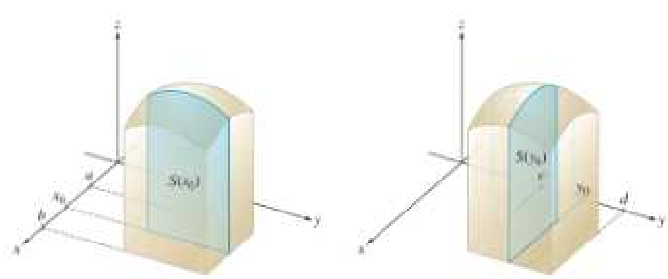
$$\int_2^4 40e^x \, dx$$

$$40[e^x]_2^4 = 40\,(e^4 - e^2)$$

Fubini's Theorem: states that an iterated integral of a continuous function $f(x, y)$ is equal with respect to either order of integration

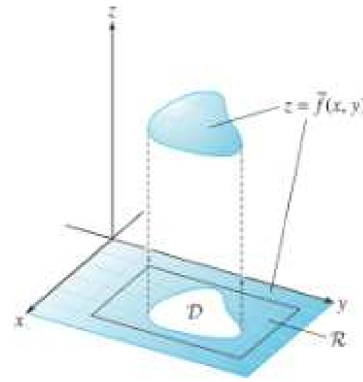
- A graphical proof can be seen below

$$\int_a^b \left(\int_c^d f(x,y) \; dy\right) dx = \int_c^d \left(\int_a^b f(x,y) \; dx\right) dy$$



HIGHLIGHT #1: Double Integrals in Non-Rectangular Regions

When the domain is non-rectangular, we use D to define the domain of the region we want to integrate over in the x-y plane.



The most difficult part of double integrals is determining the bounds for your integrals. One set of bounds will be defined as constants and the other is defined using in terms of the other variable.

There are two ways to define the bounds of a region:

Vertically Simple: the region is defined by the bounds of x and equations in terms of x (top – bottom)

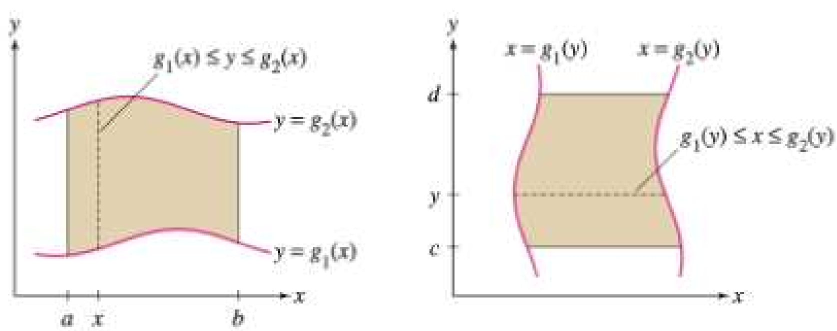
$$D = \{(x, y)\colon a \le x \le b,\ \ g_1(x) \le y \le g_2(x)\}$$

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

Horizontally Simple: the region is defined by the bounds of y and equations in terms of y (left – right)

$$D = \{(x, y)\colon c \le x \le d,\ \ g_1(y) \le x \le g_2(y)\}$$

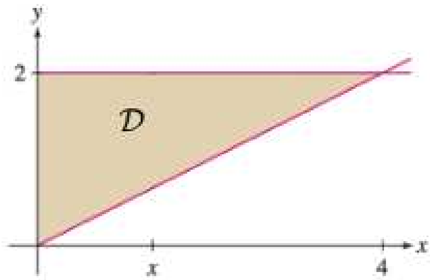
$$\int_c^d \int_{g_1(y)}^{g_2(y)} f(x, y)\, dx\, dy$$



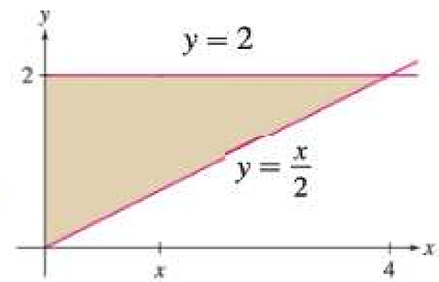
(A) Vertically simple region (B) Horizontally simple region

Choose the direction that gives a straight line!

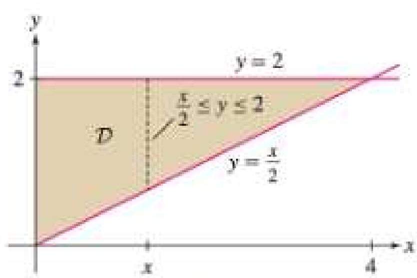
Ex. Evaluate the integral $\iint_D e^{y^2}\,dA$ for the region, D using a vertically simple and a horizontally simple domain.



1. Define the lines that bound the region



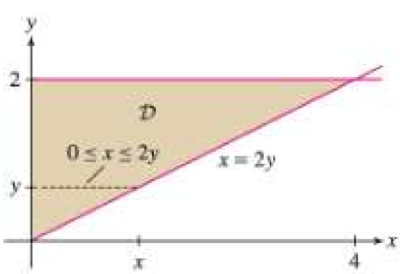
2. Describe the domain as vertically simple



$0 \le x \le 4, \quad \frac{x}{2} \le y \le 2$

$$\int_0^4 \int_{x/2}^2 e^{y^2}\,dy\,dx$$

3. Describe it as horizontally simple



$0 \le y \le 2, \quad 0 \le x \le 2y$

$$\int_0^2 \int_0^{2y} e^{y^2}\,dx\,dy$$

Average Value: the average or mean value of a function $f(x,y)$ over the domain D, can be found using the equation:

$$\bar{f} = \frac{1}{Area(D)} \iint_D f(x,y)\,dA$$

Useful videos to further explain double integrals!

Video Series on Double Integrals:
https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions#double-integrals-topic

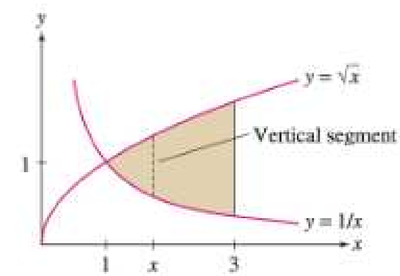
Iterated Integral Problems Worked through:
https://www.youtube.com/watch?v=czoygIbNVuM

Horizontally vs Vertically Simple Explained:
https://www.youtube.com/watch?v=3HcVfJhBPIQ

CHECK YOUR LEARNING

1. Find the volume between the graph $f(x, y) = 16 - x^2 - 3y^2$ and the rectangle $R = [0,3] \times [0,1]$.

2. Evaluate $\iint_D x^2y\, dA$ where D is the region to the right. Use a vertically simple region.



3. Find the average height $\bar{H}$ of the function $H(x, y) = 32 - x^2 - y^2$ whose base is $R = [-4,4] \times [-4,4]$.

THINGS YOU MAY STRUGGLE WITH

1. Iterated integrals are tough. It takes a lot of practice to ensure the right variables are in the right positions for horizontally and vertically simple integration. The best way to begin remembering these is to understand why the variables are where they are. Think through the integration or watch videos to help explain the order of integration.

2. Being able to evaluate double integrals is a crucial skill in Calculus 3 because it will next be applied to three dimensions. Be sure to take the time to learn and understand these. The hardest part is setting the integrals up so practice that as much as you can!

That's all for this week! I hope this was a helpful review of Chapters 15.1 – 2! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $V = 36$
2. 9
3. $\bar{H} = \frac{64}{3} \approx 21.3$