Week 8
MTH-2321 - Calculus III

Hello and Welcome to the weekly resources for MTH-2321 - Calculus III!

This week is Week 8 of class, and typically in this week of the semester. your professors are covering these topics below. If you do not see the topics your particular section of class is learning this week, please take a look at other weekly resources listed on our website for additional topics throughout of the semester.

We also invite you to look at the group tutoring chart on our website to see if this course has a group tutoring session offered this semester.

If you have any questions about these study guides, group tutoring sessions, private 30 minute tutoring appointments, the Baylor Tutoring YouTube channel or any tutoring services we offer, please visit our website www.baylor.edu/tutoring or call our drop in center during open business hours. M-Th 9am-8pm on class days 254-710-4135

Key Words: Triple Integrals, Integration in Polar/Spherical/Cylindrical Coordinates

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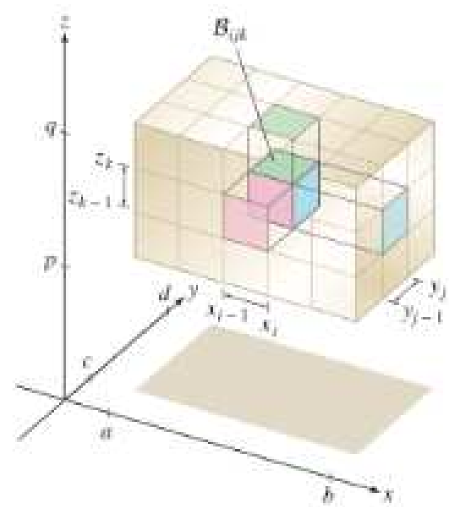
TOPIC OF THE WEEK

Triple Integrals



Triple integrals: function of $f(x, y, z)$

- Domain is represented by a box, B

$$B = [a, b] \times [c, d] \times [p, q]$$

- We subdivide the box into "sub-boxes" as seen by the colorful box in the center of the figure to the right

$$\iiint_B f(x, y, z)\, dV = \int_{x=a}^{b} \int_{y=c}^{d} \int_{z=p}^{q} f(x, y, z) dz\, dy\, dx$$

Triple integrals have the same properties and methods as single or double integrals. Like double integrals, triple integrals can be evaluated in any order and yield the same result. Solve triple integrals just like double integrals.

HIGHLIGHT #1: Evaluation Techniques for Triple Integrals

Divide the Function by Variable

If the function $f(x,y,z)$ can be divided into $g(x)h(y)k(z)$, then the triple integral is equal to the product of the individual integral of each variable

Ex. $f(x,y,z) = x^2e^{y+3z}$ could be rewritten as $x^2e^ye^{3z}$ then,

$$\iiint_B f(x,y,z)\,dV = \left(\int_a^b g(x)dx\right)\left(\int_c^d h(y)dy\right)\left(\int_p^q k(z)dz\right)$$

Using Projections

W is the region between two surfaces z_1 and z_2 in the domain D.
Therefore, the domain D is the projection of W on the xy-plane.

- W is a z-simple region
- We can use this to evaluate triple integrals as iterated integrals.

$$\iiint_W f(x,y,z)\,dW = \iint_D \left(\int_{z=z_1(x,y)}^{z=z_2(x,y)} f(x,y,z)\,dz)\right) dA$$

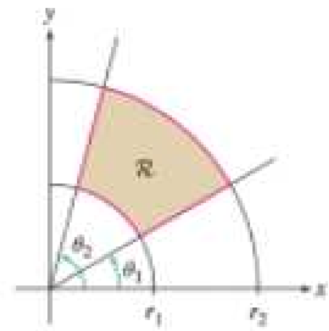
If projecting on the xy plane, the order of integration will be dz dy dx. Similarly, a projection on the yz-plane will have an order of dx dz dy. Therefore, the variable not on the projection plane is integrated first.

Average Value

The average value of a function with three variables is:

$$\bar{f} = \frac{1}{Volume(W)} \iiint_W f(x,y,z)\,dV$$

HIGHLIGHT #2: Double Integration in Polar Coordinates



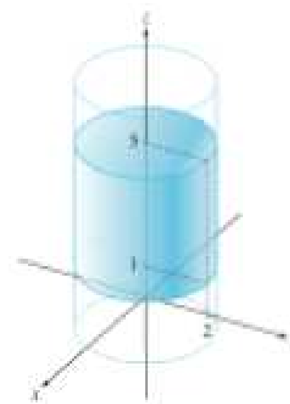
Polar coordinates are most useful for <u>polar rectangles</u> (see figure on the right)

Recall that, $x = rcos\theta$ and, $y = rsin\theta$. Therefore, the double integral can be written as:

$$\iint_R f(x,y)dA = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(rcos\theta, rsin\theta)\, r\, dr\, d\theta$$

Don't forget this additional r!

HIGHLIGHT #3: Triple Integration in Cylindrical & Spherical Coordinates

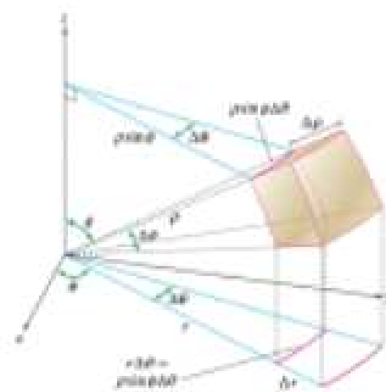


<u>Cylindrical:</u> useful for surfaces with axial symmetry

$$x = rcos\theta \qquad y = rsin\theta \qquad z = z$$

Triple integrals with cylindrical coordinates are evaluated using projections as seen in Highlight #1.

$$\int_{\theta_1}^{\theta_2}\int_{r=r_1(\theta)}^{r_2(\theta)}\int_{z=z_1(r,\theta)}^{z_2(r,\theta)} f(rcos\theta, rsin\theta, z)\, r\, dz\, dr\, d\theta$$



<u>Spherical:</u> useful for spherical wedges

$$x = \rho\, sin\phi\, cos\theta \qquad y = \rho\, sin\phi\, sin\theta \qquad z = \rho\, cos\phi$$

$$\int_{\theta_1}^{\theta_2}\int_{\phi=\phi_1}^{\phi_2}\int_{\rho=\rho_1(\theta,\phi)}^{\rho_2(\theta,\phi)} f(\rho\, sin\phi\, cos\theta, \rho\, sin\phi\, sin\theta, \rho\, cos\phi)\rho^2 sin\phi\, d\rho\, d\phi\, d\theta$$

Useful videos for further explanation of triple integrals!

Integration Steps of Triple Integrals

https://www.youtube.com/watch?v=zFy-OpajEtA

Playlist of short videos with tons of Triple Integral Topics!

https://www.youtube.com/playlist?list=PLX2gX-ftPVXXbOFICCORLptPh896DuPO2

CHECK YOUR LEARNING

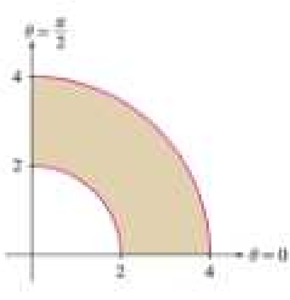
1. Calculate the integral $\iiint_B x^2 e^{y+3z}\, dV$ where $B = [1,4] \times [0,3] \times [2,6]$.

2. Evaluate $\iiint_W z\, dV$ where W is the region between the planes $z = x + y$ and $z = 3x + 5y$ over the rectangle $D = [0,3] \times [0,2]$.

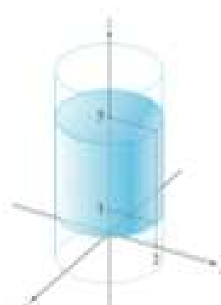
3. Compute $\iint_R (x + y) dA$ where D is a quarter annulus as seen below.

4. Integrate $f(x, y, z) = z\sqrt{x^2 + y^2}$ over the cylinder $x^2 + y^2 \le 4$ for $1 \le z \le 5$.

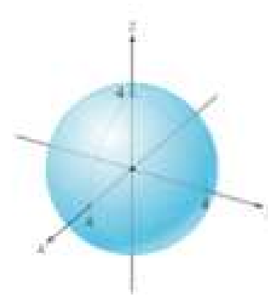
5. Integrate $f(x, y, z) = x^2 + y^2$ over the sphere S with a radius of 4, centered at the origin.



Problem #3



Problem #4



Problem #5

All pictures, tables, and information if property of Calculus Early Transcendentals (4th Edition) by Rogawski, Adams, and Franzosa.

THINGS YOU MAY STRUGGLE WITH

1. Being able to visualize or draw the regions you are looking at is the first and most difficult step in solving triple integrals. Rely on any figure given or draw your own to help you determine your bounds and the best coordinate system and method to use!

2. Don't be afraid to switch coordinate systems. While they may appear to be more challenging and more work, they are designed to make problems simpler. Feel free to start a problem in conventional rectangular coordinates and then if you find the integrals too difficult, it may be a sign you need to switch.

That's all for this week! I hope this was a helpful review of triple integrals from chapters 15.3 and 15.4! Feel free to visit the Tutoring Center Website for more information at www.baylor.edu/tutoring.

Answers:

1. $7(e^{18} - e^{6})(e^{3} - 1)$
2. 294
3. $\frac{112}{3}$
4. 64π
5. $\frac{8192\pi}{15}$