STA-1380 – Elementary Statistics
Week #4

Hello! Welcome to the additional online **Weekly Resources** for the course of **STA-1380**. Following a traditional calendar semester, these will be some of the topics your professors will go over. If you do not see material your section is going over for the week, please look at the other resources listed for this course. In addition to these resources, there might be **Group Tutoring** for this course, please see our website for more details. These sessions will go over these materials in more detail as well as any questions about the material.

Any additional help or services can be found through the **Baylor Tutoring Website**. Visit to schedule a free 30-minute private tutoring session, drop-in times for your course, the Baylor Tutoring YouTube channel, or any additional tutoring resources.

Contacts: Sid Rich M-Th 9am-8pm (Fall and Spring class days) Office Phone: 254-710-4135

Topic of the Week:
"Continuous Probability Distributions Continued"

Key Points:

- Cumulative Distribution Functions
- Uniform Distributions
- Normal Distributions

Last week's lesson introduced the continuous distribution family with the PDFs, now we will introduce two of the named continuous CDFs which are the Uniform and Normal distributions. These distributions, especially the Normal Distribution, will be used for the rest of this course and are vital for understanding how the graphs and tables of Statistics 1380 work.

Remember that for specific distributions, there will be a certain way of identifying the distribution via symbols called Function Notation often depicted as $X \sim Graph(\text{Parameters})$

The 'X' value is a representation of all values of X that the graph can depict. The 'Graph' portion of the notation is often an abbreviation of the named distribution. In last week's guides, the Binomial distribution was shortened to 'Bin' in $X \sim Bin(n,p)$. In each distribution, what goes into the parentheses will be different. What they all share is that the values listed are the steppingstones to procuring a probability. For the case of a Binomial distribution, the probability can be found by multiplying *n*p*. Look out for what each distribution's 'steppingstones' are and memorize them.

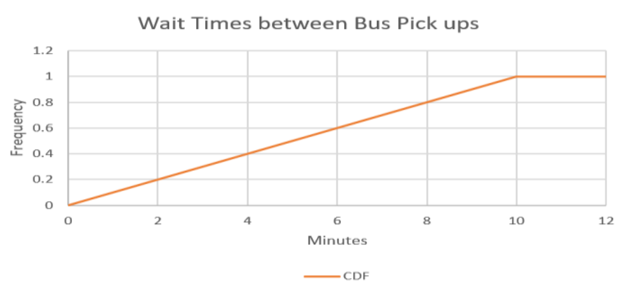
Highlight #1

"Continuous Cumulative Distribution Functions"

Definition: An accumulation of probabilities that gives the probability of having at least a set 'X' value identified by a certain and predetermined range.

Notations: $F(x) = P(X \leq x) = \int f(y)dy$

Example: Graphs, Functions, and Tables



$$F(X) = \begin{cases} 0 & X < 3 \\ \frac{3}{2}\left(\frac{x}{10}\right)^2 - \left(\frac{x}{10}\right)^3 & 3 \leq X \leq 7 \\ 1 & X > 7 \end{cases}$$

*A Uniform Distribution CDF

* A Function depicting a CDF

As mentioned in the previous weekly guide, the Cumulative Distribution Function or CDF is used for both discrete and continuous variables. However, the CDFs for continuous variables were saved for this week's lesson. An important distinction between a continuous and discrete CDF is that there is no difference between $P(X < x)$ and $P(X \leq x)$. Since $P(X = x)$ has no value for a PDF, the CDF does not consider whether the value is included and instead goes to the infinitely closest option.

A CDF's value is not derived from the area under a curve and is instead the exact value that corresponds with its point. For example, on the Bus time graph, 50% of the probability happens at 5 minutes, meaning that half of the buses can be expected to take at most 5 minutes between rotations.

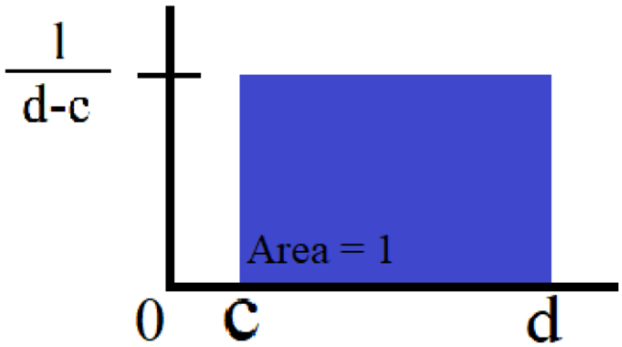
The most common type of CDF is that of the 'Table'. A very common CDF table will be shown with the Normal Distributions Highlight. These give accurate depictions of what the cumulative probability is down to 100^{th} in specifications. Tables like these allow for detailed and accurate reads rather than having to 'eyeball' a graph or calculate the results of a function. Because of its simplicity, tables will be used throughout all STA-1380 course work. A very common CDF table will be shown with the Normal Distributions Highlight.

Highlight #2
"Uniform Distribution"

Definition: A single bar graph that depicts a continuous probability across all its spanned range.

Notation: *X ~ Unif(c, d)* $f(x) = \frac{1}{d-c}$ $c \leq X \leq d$ $\mu = \frac{c+d}{2}$ $\sigma = \frac{d-c}{\sqrt{12}}$

Example: Graphs



* The PDF graph of a Uniform Distribution

The Uniform Distribution is a simple introduction into the world of continuous distributions. Often depicted in its PDF form as seen above, the graph depicts a streamline set of equal probabilities across its entire range. This range will almost always be given in the form of (c,d) in which the value of 'c' represents the absolute minimum a value within the distribution can take and 'd' the absolute maximum.

Unlike the other functions, the Uniform distribution's style of representing data is very intuitive. In order to keep the probability equal to 1, the area of the graph, which in this case is a rectangle, must also be equal to 1. Since the formula of a rectangles area can be given by Base*Height, these values must be reciprocal. If the length of the base between c and d is 4, then the height will be 1/4th. As such, at least one of these values will always be given in order to solve a problem. The height of the graph is also equal to the probability of each 'x' value occurring.

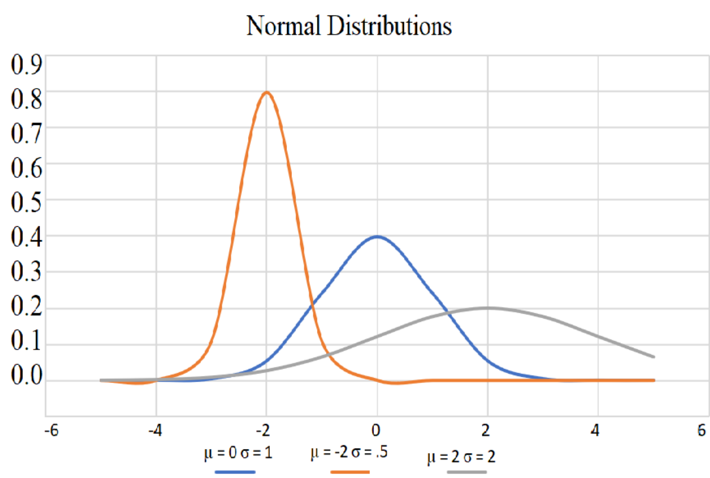
The mean of a Uniform Distribution will always be the average sum of the c and d values, the mid-point method was given above. To figure out where the std. deviation formula comes from would require calculus. For now, stick to memorizing the formula for an easier application.

Highlight #3
"Normal Distribution"

Definition: A continuous distribution with a unimodal symmetrically bell-shaped curve centered at the population's mean and shaped by the population's standard deviation normalized by the Empirical rule.

Notation: $X \sim N(\mu,\sigma)$ $\quad Z = \frac{x-\mu}{\sigma} \quad Z_\alpha = P(X \geq \alpha)$

Example: PDF Graphs and CDF Tables



* Three Normal Distribution PDF Graphs

* A Standardized Normal CDF Table

	Z-Score Normal Distribution CDF									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

Without a doubt the most important distribution in STA-1380, the Normal Distribution stands above the rest with its versatility and practical inference applications. This distribution uses an entire population's mean and standard deviation to accurately display the spread of the individuals. From frog weights to the proportion of females in a workplace, the bell-shaped curve is a perfect distribution form for most types of continuous single variable data sets.

Remember that the distribution uses the population mean and standard deviation or the parameters of a population. These are set values and are often unknown in the real world but will often be given in this course. These parameters are gained through censuses and other means of polling larger masses. If you see the symbols Mu or Sigma, (μ or σ), then intuitively, that will be your way of recognizing that the data that has been given will be a Normal Distribution.

As shown above with the PDF graphs, every Normal Distribution will be centered at the mean of the data, this is why you may also hear the mean be called the 'location' parameter. The varying 'strechiness' of a graph is determined by its 'shape' parameter or the standard deviation. A smaller std. dev. will result in a higher peak with steep slopes and tiny tails. A larger std. dev. will have wider tails and a lower peak. Remember that the term 'tails' is in reference to one or both ends of a distribution that tapper off away from the mean.

In order to properly gage the probabilities of a normal distribution and form a table, a process called the Emperical Rule is used on all Normal Distributions to 'Standardize' them. The process creates a 'Z-score,' which is the equivalent to a certain amount of standard deviations below or above the mean. The process uses 3 data points: The shape parameter, location parameter, and test statistic.

Recall that μ is the location parameter and σ is the shape parameter. The test statistic, '*x*' in this example, is what you are 'testing' for. Since the statistic can be changed, this is your variable. Do not confuse x̄ with *x* in later chapters. You can use a sample mean's results as x against the population mean, but x̄ will also function as a parameter in later distributions. Use caution and properly analyze what the question is asking before answering.

Using the second formula listed in the notation section of the guides, the standardization process is quick and simple. For this example, look at the first of the PDF graphs. It has a μ value of -2 and a standard deviation of .5. In this example, we wish to find what the probability is of having a value of -1 or fewer on this distribution. -1 would be the test statistic '*x'* for this example. Using the formula and the data points given, we would calculate the Z-score as such:

$$Z = \frac{{}^{-}1 - {}^{-}2}{.5} = \frac{1}{.5} = 2\ \sigma$$

The Emperical Rule standardizes every Normal distribution into the distribution $X \sim N(0,1)$. Every value that is produced from the standardization is a Z-score, which can be read as "_ standard deviations away from the mean." That means that a Z-score of 2.00 will be 2.00 standard deviations away from the mean of 0. Using the CDF table, the probability of having less than 2 standard deviations away from the mean is equal to .9772 or 97.7%. If you are given the probability, you can check a CDF table to find the nearest Z-score. If you are given the Z-score and one of the data points, you can solve for what the original

Typically, a Normal distribution CDF will show incremental additions to the probability starting from -∞ and going up until the decided z-score. However, there is an alternative form of the Z-score called Z-alpha, this is the first symbol in the 3rd formula, that shows probabilities from the decided z-score up to +∞ (Or reading from right to left). Graphically, a normal z-score will start at the left tail and go towards the point, while Z-alpha will start at the point and go towards the right tail. Z-alpha is meant to be a Z-score that has an 'alpha' amount of area to the right of the point. Z.0228 would have 2.28% of the probability to the right of it.

There are two ways to find the Z-scores associated with Z-alpha with a CDF table: Find the Z-score associated with the Probability of (1-alpha) or, find the Z-score associated with $P(X < \text{alpha})$. For example. If I wanted to find the Z-score associated with $Z_{.05}$, I can look to the CDF table and find the Z-score associated with .9500. The Z-score closest to this would be 1.64. The other method takes advantage of the fact that the Normal Distribution is a bell curve and that the probabilites on one side of the curve is equal to the other side. Using a CDF calculator, the Z-score associated with a probabiltiy of .05 is -1.64. When using the second method, remember to switch the negative sign around before reporting your answer.

There are many ways to find the values of a CDF. Your professor may use JMP and have the computer calculate it. Other professors may ask you to use websites or a graphing calculator with statistic functions. A calculator or website will give you a more accurate Z-score or Z-alpha. However, if you are using the CDF table and the value you are looking for is inbetween two points, pick the point with less probability. It is always better to round and loose some of the probability than risk claiming that there is a higher probability for an event than there actually is. If your professor allows you to chose between methods, use whichever statistical tool is the easiest for you to use.

Check Your Learning

1. After the 20X0 census, the state of Texas found that the average number of people living in a household was 4.2 with a standard deviation of .7.

    a. Write the Function Notation of this data set.

    b. Use the Empirical Rule to find the Z-score of a household with 5 people.

2. Avery is 5 foot 10 inches, or 70 inches, and she is in the 85^{th} percentile of her STA-1380 class.

    a. If the mean height of her course was 5'5", what is standard deviation?

    b. Using the Standard deviation, you found in Part A and find the probability associated with the heights between 5'2" and 5'9".

Things Students Struggle With

1. Converting words into Z-scores

    a. Whenever you are working through a statistics problem, A large portion of what the problem is asking you to solve for will be in words. Every question dealing with probability will have 5 parts.

    Probability, 'X', Sign, Z-score, and Equals. One way I learned how to set up these parts of the problem was the phrase, "P Exercised into Q" Phonetically, the word 'Exercise' is the combination of the sounds of "XSZ" This clue helps set up the nomenclature of $P(X < Z) =$. Always read the question and find out what the P and XSZ trio are.

    First is the Parameters or 'P' of the problem. Since this clue revolves around the fact that the question is asking for probability, there will have to be parameters set. These will be the Population standards and will be the means and standard deviations of the data set.

    Next is to find what the range of 'X' is for the problem. Does it include a tail? Between two points in the curve? Look for the range of 'X' first.

    This range will help you determine the Sign or 'S' of the problem. Is the question asking for greater than or less than the z-score? Is it between two Z-values?

Finally, use the range of 'X' as the statistic and the parameters found from 'P' to find the Z-score for the probability.

After all the parts of the problem have been identified, use a CDF to find the probability that equals, 'Q' the Z-score you have found.

2. Statistic vs Parameter:

   a. As mentioned in the Normal Distribution Highlight, the parameters are the population standards. These are expected and a single value. The statistic is the term to describe the '*x*' value that you are testing for. This value can be any value that '*X'* can take on. A sample's mean value can be chosen as the statistic and can be compared to the population mean, but a sample mean itself is the parameter of the sample. Take precautions and learn to identify whether a piece of information is a statistic or parameter.

Concluding Comments

That's it for this week! Please reach out if you have any questions and don't forget to visit the Tutoring Center website for further information at https://www.baylor.edu/tutoring.

Answers to CYL

1. **a.** *X ~ N(4.2,.7)*
   b. Z-score = 1.14

2. **a.** 5.88 standard deviations.
   b. *P(X = -.51 < x < .68)* CDF: .7517 - .305 = .4467 or 44.67%